

Fragmenting protostellar discs: properties and observational signatures

Eduard I. Vorobyov,^{1,2★} Olga V. Zakhozhay^{3★} and Michael M. Dunham⁴

¹*Institute of Astrophysics, University of Vienna, Vienna 1180, Austria*

²*Research Institute of Physics, Southern Federal University, Rostov-on-Don 344090, Russia*

³*Main Astronomical Observatory, National Academy of Sciences of Ukraine, Kyiv 03680, Ukraine*

⁴*Department of Astronomy, Yale University, PO Box 208101, New Haven, CT 06520, USA*

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ABSTRACT

Using numerical hydrodynamic simulations, we study the gravitational fragmentation of an unstable protostellar disc formed during the collapse of a pre-stellar core with a mass of $1.2 M_{\odot}$. The forming fragments span a mass range from about a Jupiter mass to very low mass protostars and are located at distances from a few tens to a thousand au, with a dearth of objects at $\lesssim 100$ au. We explore the possibility of observational detection of the fragments in discs viewed through the outflow cavity at a distance of 250 pc. We demonstrate that one hour of integration time with the Atacama Large Millimeter/sub-millimeter Array (ALMA) is sufficient to detect the fragments with masses as low as $1.5 M_{\text{Jup}}$ at orbital distances up to 800 au from the protostar. The ALMA resolution sets the limit on the minimum orbital distance of detectable fragments. For the adopted resolution of our simulated ALMA images of 0.1 arcsec, the fragments can be detected at distances down to 50 au. At smaller distances, the fragments usually merge with the central density peak. The likelihood for detecting the fragments reduces significantly for a lower resolution of 0.5 arcsec. Some of the most massive fragments, regardless of their orbital distance, can produce characteristic peaks at $\approx 5 \mu\text{m}$ and hence their presence can be indirectly inferred from the observed spectral energy distributions of protostars.

Key words: accretion, accretion discs – hydrodynamics – instabilities – stars: formation.

1 INTRODUCTION

Observations of young protostellar discs and numerical modelling of disc formation and evolution both suggest that protostellar discs may be sufficiently massive to undergo gravitational fragmentation in the course of their evolution. Observations of young protostars embedded in parental cloud cores yield disc masses that sometimes exceed $0.1 M_{\odot}$ (e.g. Isella, Carpenter & Sargent 2009; Jorgensen et al. 2009). Numerical hydrodynamics simulations indicate that such massive discs are gravitationally unstable and may fragment if disc cooling is sufficiently fast (e.g. Johnson & Gammie 2003; Stamatellos & Whitworth 2009; Vorobyov & Basu 2010a; Meru & Bate 2011; Boss 2012; Vorobyov 2013).

The newly formed fragments are in the sub-stellar mass regime and may be considered as embryos of giant planets and brown dwarfs, which may or may not evolve into finished objects. The majority of the fragments are torqued into the disc inner regions and probably on to the burgeoning protostar, triggering episodic

accretion and luminosity bursts (Vorobyov & Basu 2006, 2010a), a phenomenon supported by many indirect lines of evidence (see e.g. Dunham & Vorobyov 2012). Some of the fragments may be ejected via many-body gravitational interaction with other fragments into the intracluster medium, forming upon contraction a population of freely floating brown dwarfs (Basu & Vorobyov 2012), or settle on to stable orbits providing a gateway for the formation wide-orbit gas giant and brown dwarf companions to low-mass stars (Vorobyov & Basu 2010b; Boss 2011, 2012; Vorobyov 2013). Finally, fragments may lose their atmospheres via tidal striping and reveal solid terrestrial or icy giant cores if the dust sedimentation time-scale is sufficiently fast (Boley et al. 2010; Nayakshin 2010).

Although the feasibility of disc fragmentation has been confirmed by many theoretical and numerical studies, unambiguous observational detection of the fragments has been a challenging task. Recent observations using aperture masking interferometry have apparently been successful in identifying one such candidate (Kraus & Ireland 2012), although it might have already collapsed to a planetary-sized object as implied by the estimated age of the central star of 2 Myr. On the other hand, numerical hydrodynamic simulations seem to indicate that fragments with typical sizes of several tens of au can be

* E-mail: eduard.vorobiev@univie.ac.at (EIV); zkolga@mail.ru (OVZ)

detected with the IRAM Plateau de Bure interferometer (Stamatellos et al. 2011a). In addition, the spiral structure in gravitationally unstable discs and fragmentation on scales of the order of thousands au in starless and protostellar cores is likely to be detectable with the Atacama Large Millimeter/sub-millimeter Array (ALMA) (Cossins, Lodato & Testi 2010; Offner et al. 2012a).

In this paper, we study numerically the formation and evolution of a protostellar disc subject to vigorous gravitational fragmentation in the embedded phase of star formation. We focus on the properties of the fragments and explore possibilities of their direct and indirect observational detection. In Section 2, an overview of the numerical model is given. Section 2.3 describes the adopted model for calculating the spectral energy distributions (SEDs). The main properties of the fragments are presented in Sections 3 and 4. The model SEDs are discussed in Section 5 and the synthetic ALMA images are presented in Section 6. The limitations of our model are discussed in Section 7 and main conclusions are given in Section 8.

2 MODEL DESCRIPTION AND BASIC EQUATIONS

2.1 Numerical hydrodynamics model

Our numerical hydrodynamics model for the formation and evolution of a star plus disc system is described in detail in Vorobyov & Basu (2010a) and is briefly reviewed below for the reader's convenience. We start our numerical simulations from the gravitational collapse of a *starless* cloud core, continue into the embedded phase of star formation, during which a star, disc and envelope are formed, and terminate our simulations in the T Tauri phase, when most of the envelope has accreted on to the forming star/disc system. In our numerical hydrodynamics simulation, the protostellar disc is not isolated but is exposed to intense mass loading from the envelope. In addition, the mass accretion rate on to the disc is not a free parameter of the model but is self-consistently determined by the gas dynamics in the envelope, which in turn is set by initial conditions in the core.

To avoid too small time steps, we introduce a 'sink cell' at $r_{\text{sc}} = 7 \text{ au}$ and impose a free inflow inner boundary condition and free outflow outer boundary condition so that the matter is allowed to flow out of the computational domain but is prevented from flowing in. The sink cell is dynamically inactive; it contributes only to the total gravitational potential and secures a smooth behaviour of the gravity force down to the stellar surface. We monitor the gas surface density in the sink cell and when its value exceeds a critical value for the transition from isothermal to adiabatic evolution, we introduce a central point-mass object. In the subsequent evolution, 90 per cent of the gas that crosses the inner boundary is assumed to land on to the central object plus the sink cell. The other 10 per cent of the accreted gas is assumed to be carried away with protostellar jets. We assumed a 10 per cent efficiency as the typical lower limit in the X-wind model of jet launching (Shu et al. 1994).

We make use of the thin-disc approximation to compute the gravitational collapse of rotating, gravitationally unstable cloud cores. This approximation is an excellent means to calculate the evolution for many orbital periods and many model parameters and its justification is discussed in Vorobyov & Basu (2010a). The basic equations of mass, momentum and energy transport are

$$\frac{\partial \Sigma}{\partial t} = -\nabla_p \cdot (\Sigma \mathbf{v}_p), \quad (1)$$

$$\frac{\partial}{\partial t} (\Sigma \mathbf{v}_p) + [\nabla \cdot (\Sigma \mathbf{v}_p \otimes \mathbf{v}_p)]_p = -\nabla_p \mathcal{P} + \Sigma \mathbf{g}_p + (\nabla \cdot \mathbf{\Pi})_p, \quad (2)$$

$$\frac{\partial e}{\partial t} + \nabla_p \cdot (e \mathbf{v}_p) = -\mathcal{P}(\nabla_p \cdot \mathbf{v}_p) - \Lambda + \Gamma + (\nabla \mathbf{v})_{pp'} : \Pi_{pp'}, \quad (3)$$

where subscripts p and p' refers to the planar components (r, ϕ) in polar coordinates, Σ is the mass surface density, e is the internal energy per surface area, \mathcal{P} is the vertically integrated gas pressure calculated via the ideal equation of state as $\mathcal{P} = (\gamma - 1)e$ with $\gamma = 7/5$, Z is the radially and azimuthally varying vertical scale-height determined in each computational cell using an assumption of local hydrostatic equilibrium, $\mathbf{v}_p = v_r \hat{\mathbf{r}} + v_\phi \hat{\boldsymbol{\phi}}$ is the velocity in the disc plane, $\mathbf{g}_p = g_r \hat{\mathbf{r}} + g_\phi \hat{\boldsymbol{\phi}}$ is the gravitational acceleration in the disc plane and $\nabla_p = \hat{\mathbf{r}} \partial / \partial r + \hat{\boldsymbol{\phi}} r^{-1} \partial / \partial \phi$ is the gradient along the planar coordinates of the disc.

Turbulent viscosity is taken into account via the viscous stress tensor $\mathbf{\Pi}$, the expression for which is provided in Vorobyov & Basu (2010a). We parametrize the magnitude of kinematic viscosity ν using a modified form of the α -prescription

$$\nu = \alpha c_s Z \mathcal{F}_\alpha(r), \quad (4)$$

where $c_s^2 = \gamma \mathcal{P} / \Sigma$ is the square of effective sound speed calculated at each time step from the model's known \mathcal{P} and Σ . The function $\mathcal{F}_\alpha(r) = 2\pi^{-1} \tan^{-1}[(r_d/r)^{10}]$ is a modification to the usual α -prescription that guarantees that the turbulent viscosity operates only in the disc and quickly reduces to zero beyond the disc radius r_d . In this paper, we use a spatially and temporally uniform α , with its value set to 5×10^{-3} . This choice is based on the work of Vorobyov & Basu (2009), who studied numerically the long-term evolution of viscous and self-gravitating discs. They found that the temporally and spatially averaged α in protostellar discs should lie in the 10^{-3} – 10^{-2} limits. Smaller values of α ($\leq 10^{-4}$) have little effect on the resultant mass accretion history (dominated in this case by gravitational torques), while larger values ($\alpha > 10^{-1}$) destroy circumstellar discs during less than 1.0 Myr of evolution and are thus inconsistent with mean disc lifetimes of the order of 2–3 Myr.

The radiative cooling Λ in equation (2) is determined using the diffusion approximation of the vertical radiation transport in a one-zone model of the vertical disc structure (Johnson & Gammie 2003)

$$\Lambda = \mathcal{F}_c \sigma T_{\text{mp}}^4 \frac{\tau}{1 + \tau^2}, \quad (5)$$

where σ is the Stefan–Boltzmann constant, $T_{\text{mp}} = \mathcal{P} \mu / R \Sigma$ is the mid-plane temperature of gas,¹ $\mu = 2.33$ is the mean molecular weight, R is the universal gas constant and $\mathcal{F}_c = 2 + 20 \tan^{-1}(\tau) / (3\pi)$ is a function that secures a correct transition between the optically thick and optically thin regimes. We use frequency-integrated opacities of Bell & Lin (1994). The heating function is expressed as

$$\Gamma = \mathcal{F}_c \sigma T_{\text{irr}}^4 \frac{\tau}{1 + \tau^2}, \quad (6)$$

where T_{irr} is the irradiation temperature at the disc surface determined by the stellar and background blackbody irradiation as

$$T_{\text{irr}}^4 = T_{\text{bg}}^4 + \frac{F_{\text{irr}}(r)}{\sigma}, \quad (7)$$

¹ This definition of the mid-plane temperature is accurate within a factor of unity (Zhu et al. 2012).

where T_{bg} is the uniform background temperature (in our model set to the initial temperature of the natal cloud core) and $F_{\text{irr}}(r)$ is the radiation flux (energy per unit time per unit surface area) absorbed by the disc surface at radial distance r from the central star. The latter quantity is calculated as

$$F_{\text{irr}}(r) = \frac{L_*}{4\pi r^2} \cos \gamma_{\text{irr}}, \quad (8)$$

where γ_{irr} is the incidence angle of radiation arriving at the disc surface at radial distance r calculated as (Vorobyov & Basu 2010a)

$$\cos \gamma_{\text{irr}} = \cos \alpha \cos \delta (\tan \alpha - \tan \delta), \quad (9)$$

where

$$\cos \alpha = \frac{dr}{\sqrt{dr^2 + dZ^2}}; \quad \cos \delta = \frac{r}{\sqrt{r^2 + Z^2}}, \quad (10)$$

$$\tan \alpha = \frac{dZ}{dr}; \quad \tan \delta = \frac{Z}{r}. \quad (11)$$

The stellar luminosity L_* is the sum of the accretion luminosity $L_{*,\text{accr}} = GM_*\dot{M}/2R_*$ arising from the gravitational energy of accreted gas and the photospheric luminosity $L_{*,\text{ph}}$ due to gravitational compression and deuterium burning in the star interior. The stellar mass M_* and accretion rate on to the star \dot{M} are determined self-consistently during numerical simulations via the amount of gas passing through the sink cell. The stellar radius R_* is calculated using an approximation formula of Palla & Stahler (1991), modified to take into account the formation of the first molecular core. The photospheric luminosity $L_{*,\text{ph}}$ is taken from the pre-main-sequence tracks for the low-mass stars and brown dwarfs calculated by D'Antona & Mazitelli (1997). More details are given in Vorobyov & Basu (2010a).

Equations (1)–(2) are solved using the method of finite differences with a time explicit solution procedure in polar coordinates (r, ϕ) on a numerical grid with 512×512 grid zones. The advection is treated using a third-order-accurate piecewise parabolic interpolation scheme of Colella & Woodward (1984). The update of the internal energy per surface area e due to cooling Λ and heating Γ is done implicitly using the Newton–Raphson method of root finding, complemented by the bisection method where the Newton–Raphson iterations fail to converge. The viscous heating and force terms in equations (2) and (3) are implemented in the code using an explicit finite-difference scheme, which is found to be adequate for $\alpha \lesssim 0.01$. The radial points are logarithmically spaced. The innermost grid point is located at the position of the sink cell $r_{\text{sc}} = 7$ au, and the size of the first adjacent cell varies in the 0.07–0.1 au range depending on the cloud core size. This corresponds to the radial resolution of $\Delta r = 1.1$ –1.6 au at 100 au.

2.2 Initial setup

For the initial distribution of the gas surface density Σ and angular velocity Ω in the pre-stellar cores, we take those from Basu (1997)

$$\Sigma = \frac{r_0 \Sigma_0}{\sqrt{r^2 + r_0^2}}, \quad (12)$$

$$\Omega = 2\Omega_0 \left(\frac{r_0}{r}\right)^2 \left[\sqrt{1 + \left(\frac{r}{r_0}\right)^2} - 1 \right]. \quad (13)$$

These radial profiles are typical of pre-stellar cores formed as a result of the slow expulsion of magnetic field due to ambipolar diffusion,

with the angular momentum remaining constant during axially symmetric core compression. Here, Ω_0 and Σ_0 are the angular velocity and gas surface density at the disc centre and $r_0 = \sqrt{Ac_s^2/\pi G \Sigma_0}$ is the radius of the central plateau, where c_s is the initial sound speed in the core. The gas surface density distribution described by equation (12) can be obtained (to within a factor of unity) by integrating the three-dimensional gas density distribution characteristic of Bonnor–Ebert spheres with a positive density-perturbation amplitude A (Dapp & Basu 2009). In all models the value of A is set to 1.2. We note that the properties of protostellar discs formed as a result of the gravitational collapse of pre-stellar cores depend weekly on the form of the initial density and angular velocity distributions in the core as long as the initial core mass and ratio of rotational to gravitational energy β are the same (Vorobyov 2012). The initial temperature of pre-stellar cores is set to 10 K.

2.3 Constructing spectral energy distributions

To construct an SED of our model object, we split it into three constituent parts: the accreting protostar, the inner disc and the outer dynamic disc.² This partitioning is motivated by the specifics of our numerical model – we do not compute the dynamical evolution of the system in the inner several au (inside the sink cell) due to severe Courant limitations on the hydrodynamical time step.

The basic assumption entering our calculation of the SEDs is that our model object is viewed through an outflow cavity. This allows us to neglect the reprocessing of light by the envelope and use surface densities and effective temperatures derived directly from numerical hydrodynamics modelling of the outer disc and analytic model of the inner disc. At the same time, this assumption imposes limitations on the viewing angle, which should not exceed the opening semi-angle of the outflow cavity θ_c . Below, we explain in more detail the main ideas and equations involved in calculating the model SEDs.

2.3.1 The outer dynamic disc

Numerical hydrodynamics simulations provide us with gas surface densities Σ and mid-plane temperatures T_{mp} in each grid cell (i, j) of the outer dynamic disc. We use a solution to the one-dimensional radiation transfer equation to calculate the radiative flux density F_ν leaving each grid cell of the disc surface

$$F_\nu^{\text{disc}} = \frac{S}{d^2} B_\nu(T_{\text{surf}}) (1 - e^{-\Sigma \kappa_\nu \sec \gamma_{\text{incl}}}), \quad (14)$$

where $B_\nu(T_{\text{surf}})$ is the Planck function, d is the distance to the considered object, set to 100 pc in the paper, γ_{incl} is the inclination angle of the disc with respect to the observer (i.e. an angle between the disc rotation axis and the line of sight) and S is the projected area of the disc surface seen by the observer. The inclination angle γ_{incl} is set to zero in this paper. The effect of non-zero inclination, but not exceeding the opening semi-angle of the outflow cavity, is discussed in Section 7 and will be considered in detail in a follow-up paper. The total flux is found by summing inputs from all individual grid cells of the disc, which amounts to 512×512 zones for the typical numerical resolution of our hydro simulations.

² The outer dynamic disc may include an infalling envelope, if present. Because it is often difficult to draw a clear distinction between the disc and envelope, we refer to both simply as the outer disc.

We adopt frequency-dependent opacities κ_ν from Ossenkopf & Henning (1994). These opacities were derived for three cases of (a) thick ice mantles, (b) thin ice mantles and (c) no ice mantles. In this work, we adopt opacities derived for the case of thin ices, commonly referred to as ‘OHS’ opacities in the literature. The dependence of our results on the adopted opacity is discussed in more detail in Section 7.

To calculate the Plunk function B_ν in equation (14), we need to know the disc surface temperature T_{surf} . Stellar/background irradiation of the disc surface, viscous/shock heating of the disc interiors and blackbody cooling from the disc surface determine the temperature balance of the disc in our model. Therefore, we use the following equation to calculate the surface temperature of the disc in our model

$$T_{\text{surf}}^4 = \frac{1}{2} \mathcal{F}_c T_{\text{mp}}^4 \frac{1}{1 + \tau} + T_{\text{bg}}^4 + \frac{F_{\text{irr}}(r)}{\sigma}, \quad (15)$$

where the factor 1/2 is introduced because only the fluxes reaching/leaving one side of the disc are considered. We also note that the effect of disc opacity is now taken into account via the factor $1/(1 + \tau)$ rather than via $\tau/(1 + \tau^2)$. The latter expression yields too low temperatures in the optically thin limit $\tau < 1.0$ in cases when the heating sources are absent.

2.3.2 The central accreting protostar

The central protostar provides a significant contribution to the SED at shorter wavelengths. The radiative flux density from the protostar F_ν^* is calculated assuming a blackbody radiation spectrum with effective temperature T_{eff}^*

$$F_\nu^* = \frac{\pi R_*^2}{d^2} B_\nu(T_{\text{eff}}^*), \quad (16)$$

where R_* is the radius of the protostar and T_{eff}^* is defined by both the accretion and photospheric luminosities ($L_{*, \text{accr}}$ and $L_{*, \text{ph}}$, respectively) as

$$T_{\text{eff}}^* = \left(\frac{L_{\text{accr}} + L_{\text{ph}}}{4\pi R_*^2 \sigma} \right)^{1/4}, \quad (17)$$

where σ is the Stefan–Boltzmann constant. The accretion luminosity is calculated from the model’s known accretion rate \dot{M} on to the protostar, while the photospheric luminosity is taken from the pre-main-sequence tracks of D’Antona & Mazitelli (1997) for low-mass stars and brown dwarfs.

2.3.3 The inner disc

The properties of the disc in the inner several au cannot be calculated self-consistently because of the sink cell, which is introduced for numerical convenience to avoid too small timesteps near the origin where the grid lines converge. Nevertheless, the inner disc may contribute significantly to the total SED. We reconstruct the structure of the inner disc using the azimuthally averaged gas surface density profile in the outer disc and adopting a simple model for the surface temperature of the inner disc.

The reconstruction of the physical properties of the inner disc proceeds in three steps. First, we calculate the inner disc truncation radius R_{in} where the gas temperature exceeds the dust sublimation threshold $T_{\text{ds}} = 1500 \text{ K}$ (Dullemond, Dominik & Natta 2001). The effect of varying T_{ds} is discussed in Section 7. The calculation is based on the assumption of energy balance between the energy absorbed by spherical, blackbody dust grains

$\pi a^2(L_{*, \text{accr}} + L_{*, \text{ph}})/(4\pi r^2)$ and energy emitted by dust grains $\sigma T^4 4\pi a^2$, where a is the radius of dust grains (Dunham et al. 2010). The resulting expression for the sublimation radius is

$$R_{\text{in}} = \sqrt{\frac{L_{*, \text{accr}} + L_{*, \text{ph}}}{16\pi\sigma T_{\text{ds}}^4}}. \quad (18)$$

We assume that the inner rim of the disc at R_{in} is heated to the temperature of dust sublimation T_{ds} .

Secondly, we calculate the gas surface density distribution in the inner disc using the following relation

$$\Sigma_{\text{in}}(r) = \Sigma_{\text{jn},0} \left(\frac{r}{r_{\text{jn}}} \right)^{-p}, \quad (19)$$

where r_{jn} is the outer radius of the inner disc and $\Sigma_{\text{jn},0}$ is the corresponding gas surface density. To make a smooth transition between the inner and outer discs, we perform the least-squares fitting to the azimuthally averaged gas surface density of the latter. Based on this fitting, we choose a value of $\Sigma_{\text{jn},0}$ at r_{jn} such that the two discs are smoothly joined. The exponent p varies between 1.5 for gravitationally unstable discs and ≈ 1.0 for viscosity-dominated discs (e.g. Vorobyov & Basu 2009). Since the inner disc is likely to be gravitationally stable, we chose $p = 1.0$. We note however that variations in the density profile of the inner disc insignificantly influence the resulting SED due to the high optical depth of the inner disc.

Finally, the surface temperature of the inner disc is found adopting a simple model of a viscous disc in Keplerian rotation. Viscosity generates dissipation of energy in the vertical column of the disc at a rate ($\text{erg cm}^{-2} \text{ s}^{-1}$)

$$\dot{E} = \frac{9}{4} \nu \Sigma \Omega^2, \quad (20)$$

where $\Omega = (GM_*/r^3)^{1/2}$.

Assuming further that the inner disc radiates from its surface as a blackbody, one can write the disc surface temperature as

$$T_{\text{surf}}^4 = \frac{9}{8\sigma} \nu \Sigma \Omega^2 + T_{\text{bg}}^4 + \frac{F_{\text{irr}}}{\sigma}. \quad (21)$$

The inner disc scale-height Z_{in} , needed to calculate the incident radiation flux F_{irr} , is found by assuming the disc aspect ratio from D’Alessio et al. (1999)

$$\frac{Z_{\text{in}}}{r} = C \left(\frac{r}{r_{\text{jn}}} \right)^{0.25}, \quad (22)$$

where the scaling factor C is found by applying the disc scale-height of the outer dynamic disc at r_{jn} . This ensures a continuous transition between the inner and outer disc vertical scale-heights. For a steady-state disc, the dynamic viscosity $\nu \Sigma$ can be expressed in terms of the mass accretion rate \dot{M} as (Pringle 1981)

$$\nu \Sigma = \frac{\dot{M}}{3\pi} \left(1 - \left(\frac{R_*}{r} \right)^{1/2} \right). \quad (23)$$

For the mass accretion rate \dot{M} in the inner disc we adopted the value calculated at the inner inflow boundary of the outer dynamic disc, $r_{\text{sc}} = 7 \text{ au}$. Once the surface temperature and density in the inner disc have been calculated, the input to the total SED is found using equation (14). The implementation of a more accurate, time-dependent model of the inner disc, subject to mass loading from the outer dynamic disc, remains to be a future improvement of our numerical model.

3 DISC INSTABILITY AND FRAGMENTATION

Theoretical and numerical studies of the evolution of protostellar discs indicate that disc fragmentation is a complicated phenomenon, which can be influenced by both the internal disc physics and external environment. The latter may influence the disc susceptibility to fragmentation directly through, e.g. stellar and background irradiation, (e.g. Vorobyov & Basu 2010a; Stamatellos et al. 2011b) or indirectly by setting the initial conditions in cloud cores that favour or disfavour fragmentation in subsequently formed discs due to, e.g. higher/lower infall rates on to the disc (Vorobyov & Basu 2006; Kratter, Matzner & Krumholz 2008; Vorobyov 2011).

In this section, we describe the time evolution of protostellar discs formed as a result of the gravitational collapse of pre-stellar cores with different initial mass and angular momentum. We consider two prototype cases: model 1 with initial core mass $M_c = 1.22 M_\odot$ and ratio of rotational to gravitational energy $\beta = 8.8 \times 10^{-3}$ and also model 2 with $M_c = 0.74 M_\odot$ and $\beta = 2.75 \times 10^{-3}$. We have chosen the two models based on our previous numerical simulations (Vorobyov & Basu 2010a) so as to obtain discs with vigorous and occasional gravitational fragmentation.

Fig. 1 presents the gas surface density images for model 1 in the inner 2000×2000 au box at various times since the formation of the central protostar. The entire computational box is almost 10 times larger. The disc forms at $t = 0.01$ Myr and becomes gravitationally unstable at $t = 0.04$ Myr. A first set of fragments appear in the disc at $t = 0.1$ Myr. The subsequent evolution is characterized by vigorous gravitational instability and fragmentation leading sometimes to a nearly complete breakup of the disc into massive fragments and dense filamentary arms (i.e. at $t = 0.13$ Myr). Some of the fragments possess mini discs of their own, complicating even more the dynamics of the entire disc-like structure.

When studying the susceptibility of a protostellar disc to gravitational fragmentation, the following two criteria are usually investigated (e.g. Rafikov 2005; Vorobyov & Basu 2010a).

(i) The ratio of the local cooling time $t_c = e/\Lambda$ to the local dynamical time Ω^{-1} should be smaller than a few (Gammie 2001; Rice et al. 2003; Meru & Bate 2012), with the actual value depending on the physical conditions in the disc. Following Vorobyov & Basu (2010a), we will refer to the dimensionless quantity $t_c \Omega^{-1}$ as the \mathcal{G} -parameter and adopt $\mathcal{G} = 1$ as a fiducial (and conservative) critical value.

(ii) The Toomre criterion $Q = c_s \Omega / (\pi G \Sigma)$ for a Keplerian disc is smaller than some critical value Q_{cr} , usually taken to be unity (Toomre 1964). The value of Q_{cr} may depend on the physical conditions and may vary by a factor of unity (Polyachenko, Polyachenko & Strel'nikov 1997). We have chosen a conservative value of $Q_{\text{cr}} = 1.0$.

We applied these two criteria to our model at $t = 0.13$ Myr in a fashion similar to figs 3 and 4 in Vorobyov & Basu (2010a). We found that the fragments obey both criteria. At the same time, there are a few regions in the disc that are characterized by $Q < 1.0$ but with no obvious fragmentation taking place in there. The lack of fragmentation in these regions is explained by slow cooling. We also note that both criteria for disc fragmentation are violated in the inner several tens of au, in agreement with previous studies of fragmenting discs.

We note that the total mass of the disc (including fragments and the sink cell) varies between $0.19 M_\odot$ at $t = 0.09$ Myr and $0.25 M_\odot$ at $t = 0.6$ Myr, with the maximum value of $0.31 M_\odot$ reached at $t = 0.25$ Myr. The mass of the central star grows from $0.32 M_\odot$ at $t = 0.09$ Myr to $0.76 M_\odot$ at $t = 0.6$ Myr. For the disc mass estimates, we used an algorithm described in Dunham & Vorobyov (2012) and a critical surface density for the disc to envelope transition of 0.1 g cm^{-2} .

Finally, in Fig. 2 we present an example of a gravitationally unstable disc showing only occasional fragmentation. The panels show the gas surface density maps in model 2 for the inner 800×800 au box. The time elapsed since the formation of the central protostar

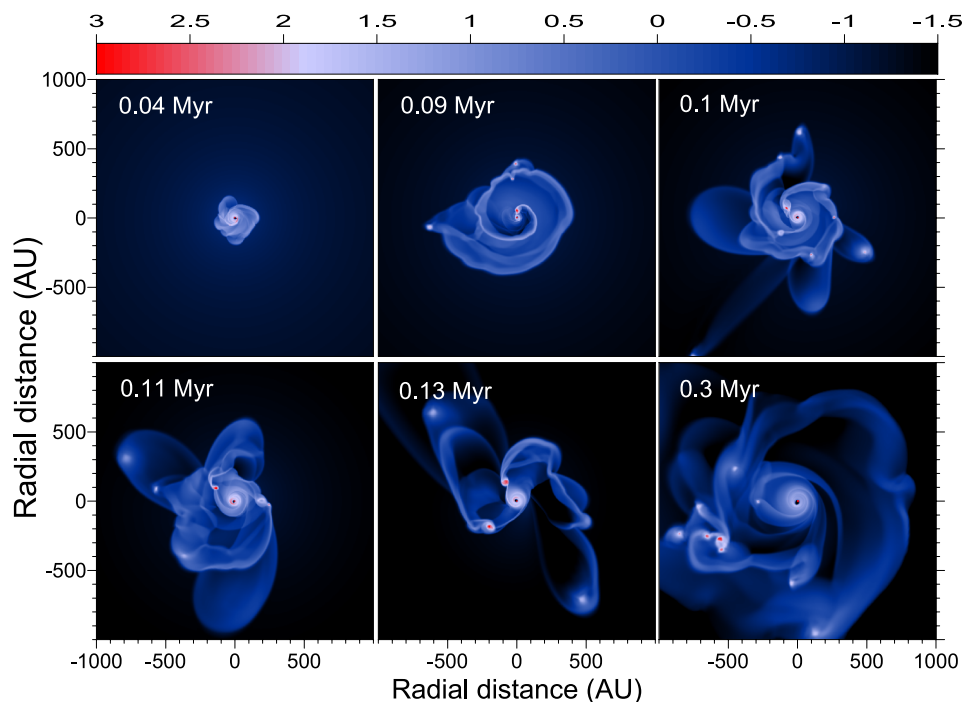


Figure 1. Gas surface density maps of the inner 2000×2000 au box in our model. The time elapsed since the formation of the central protostar is indicated in each panel. The scale bar is in $\log \text{ g cm}^{-2}$. The protostar is at the coordinate centre.

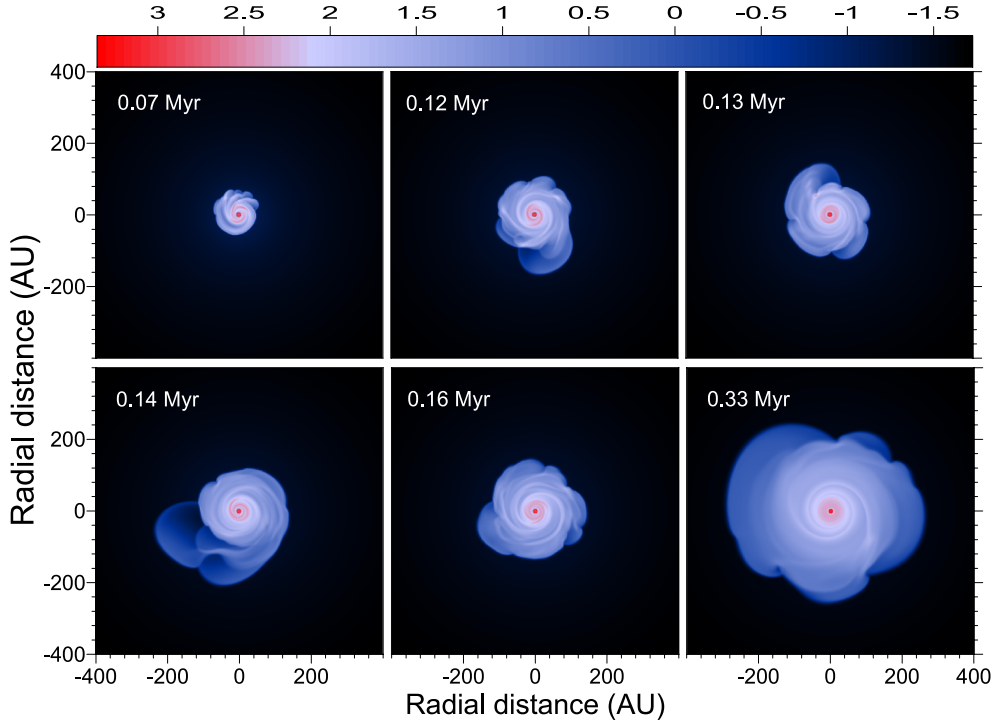


Figure 2. The same as Fig. 1 only for model 2.

is indicated in each panel. The disc in this model forms at $t = 0.035$ Myr, appreciably later than in model 1 due to smaller angular momentum of the parental pre-stellar core. The resulting disc mass is smaller by almost a factor of 2 than in model 1. The disc is gravitationally unstable and exhibits a flocculent spiral structure but is mainly stable to fragmentation, in contrast to model 1. The only distinct episode of disc fragmentation takes place around $t = 0.13$ Myr, but the fragment has quickly migrated through the inner sink cell during just several orbital periods. In the subsequent evolution, the disc shows little signs of fragmentation. We note that the disc age in each panel is approximately equal to that in the corresponding panel in Fig. 1.

4 PROPERTIES OF THE FRAGMENTS

In this section, we explore the properties of the fragments in model 1 with the purpose of determining the feasibility of using observations to indirectly or directly detect their presence. Fig. 3 shows the number of fragments N_f calculated every 0.01 Myr after the formation of the protostar using a fragment tracking algorithm described later in this section. Evidently, N_f varies significantly with time, reaching local peak values every ≈ 0.1 Myr. An increase in the number of fragments shows recent fragmentation, and a decrease shows recent destruction/accretion of the fragments.

It is interesting to note that the time intervals between recurrent peaks is of the same duration as the characteristic time of mass infall on to the disc

$$t_{\text{infall}} = \frac{M_d}{\dot{M}_d}, \quad (24)$$

where M_d is the disc mass and $\dot{M}_d \propto c_s^3/G$ is the mass infall rate on to the disc. Indeed, for the disc mass of $0.2\text{--}0.25 M_\odot$ and mass infall

rates of $(2\text{--}3) \times 10^{-6} M_\odot$, typical for our model, the characteristic infall time ranges between 0.07 and 0.12 Myr. The approximate match between periodicity of the peaks in Fig. 3 and t_{infall} suggests that the mass infall may cause periodic bursts of disc fragmentation, at least in the embedded phase of disc evolution which lasts for 0.4–0.45 Myr in this model. The important implication of this phenomenon is that the fragments can be present in the disc for a significantly longer time period than expected for just one burst of disc fragmentation in numerical hydrodynamics simulations without disc infall (e.g. Stamatellos et al. 2011a), significantly increasing the probability for observing the fragments.

We use a clump tracking mechanism described in Vorobyov (2013) to determine the position and properties of the fragments in model 1. The method is briefly outlined below. Fragments are identified on the computational mesh based on two criteria

$$\frac{\partial \mathcal{P}}{\partial r'} + \frac{1}{r'} \frac{\partial \mathcal{P}}{\partial \phi'} < 0, \quad (25)$$

$$\frac{\partial \Phi}{\partial r'} + \frac{1}{r'} \frac{\partial \Phi}{\partial \phi'} > 0, \quad (26)$$

where Φ is the gravitational potential and the primed coordinates are defined as $r' = r - r_c$ and $\phi' = \phi - \phi_c$. The polar coordinates r_c and ϕ_c denote the geometrical centre of the fragment, which is determined as the position of a local maximum in the gas surface density on the computational mesh. The first condition mandates that the fragment must be pressure supported, with a negative pressure gradient with respect to the centre of the fragment. The second condition requires that the fragment is kept together by gravity, with the potential well being deepest at the centre of the fragment. All grid cells that satisfy both equations (25) and (26), starting from the first layer of cells immediately adjacent to (r_c, ϕ_c) , are defined as belonging to the fragment.

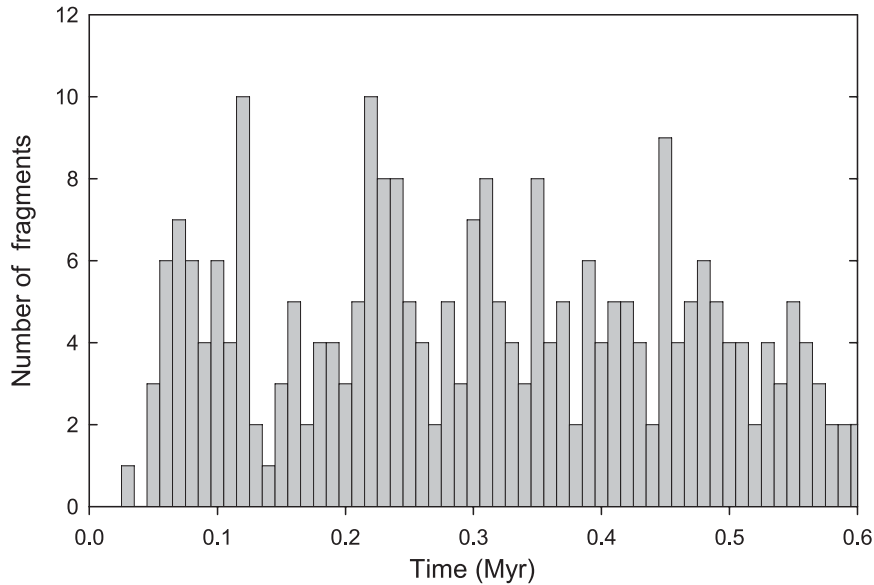


Figure 3. Number of fragments versus time in model 1. The number of fragments at a given time instant is calculated using the fragment tracking algorithm described in Section 4. An increase in the number of fragments shows recent fragmentation, and a decrease shows recent destruction/accretion of the fragments.

Panels (a) and (b) in Fig. 4 present the normalized distribution functions of fragment mass and radial positions in model 1. These distributions were calculated by searching for and identifying the fragments in the inner 5000 au of the computational mesh every 0.01 Myr starting from $t = 0.03$ Myr and ending at $t = 0.6$ Myr. The obtained masses and radial positions of the fragments were then split into 20 logarithmically spaced bins. One can identify two local maxima in the mass distribution function, one belonging to the planetary-mass regime at $\approx 5M_{\text{Jup}}$ and the other lying in the upper brown-dwarf-mass regime at $\approx 60\text{--}70M_{\text{Jup}}$. The whole mass spectrum extends from about a Jupiter mass to very low mass stars. The radial positions of the fragments have a well-defined maximum at $\approx 400\text{--}500$ au. The number of the fragments gradually declines at smaller distances and no fragments are seen at $r < 30$ au. The dearth of fragments at $r < 100$ au is most likely explained by slow cooling that impedes fragmentation in the inner several tens au (Johnson & Gammie 2003; Meru & Bate 2011) and fast inward radial migration of the fragments. Numerical simulations show that once migration has started it proceeds very fast and fragments spiral into the inner few au (and probably on to the star) during just a few orbital periods (Vorobyov & Basu 2006, 2010a; Cha & Nayakshin 2011; Machida, Inutsuka & Matsumoto 2011), making it difficult to observe such fragments. We note that we do not follow the fate of the fragments once they have passed through the sink cell at 7 au.

The two bottom panels in Fig. 4 present vertically integrated pressures \mathcal{P}_c , surface densities Σ_c , mid-plane temperatures $T_{\text{mp},c}$ and surface temperatures $T_{\text{surf},c}$ derived at the geometrical centres of the fragments in model 1. As before, these data were calculated by searching for and identifying fragments in the inner 5000 au every 0.01 Myr starting from $t = 0.03$ Myr and ending at $t = 0.6$ Myr. We emphasize that the derived data do not always correspond to distinct fragments but sometimes represent the same fragment at different evolutionary times. The solid and dashed lines in panel (c) of Fig. 4 present the least-squares fitting to the \mathcal{P}_c versus Σ_c data. In particular, the solid line provides the best fit for the fragments with $\Sigma_c < 10^3 \text{ g cm}^{-2}$, while the dashed line does that for the fragments with $\Sigma_c \geq 10^3 \text{ g cm}^{-2}$. The resulting exponents are 2.0

(solid line) and 1.55 (dashed lines), implying that the fragments can be described by a polytrope with the polytropic index $n = (1.0\text{--}2.0)$ (related to the adiabatic index as $\gamma = (n + 1)/n$). The value of n seems to depend on the density. Radiative fragments are supposed to have $n = 1.5$, while convective ones are characterized by $n = 2.5$. Since in our 2D approach the vertical convection is not taken into account, it is not surprising that our fragments are radiative. However, mixing in the disc plane may still take place for the most massive fragments, which may explain values of the polytropic index greater than 1.5.

The open circles in panel (d) of Fig. 4 show mid-plane temperature versus surface density, while the filled triangles show surface temperatures versus surface density. The surface temperatures are appreciably lower than the mid-plane ones, as can be expected for optically thick fragments. We have not identified any fragments with $T_{\text{mp},c}$ exceeding 2000 K, implying that all the fragments are the first hydrostatic cores which have not yet dissociated molecular hydrogen.

Typical disc surface densities and temperatures at $r \gtrsim 100$ au, where most fragments are located, do not, as a rule, exceed 100 g cm^{-3} and 50 K, respectively (Vorobyov 2011). Most fragments in our model are characterized by surface densities greater than 100 g cm^{-2} . Surface temperatures of the fragments are usually much lower than those at the mid-plane due to the high optical depths of the fragments. The majority of the fragments are characterized by $T_{\text{surf},c} < 100$ K. However, there are four fragments with mid-plane temperatures exceeding 10^3 K, which means that they have started evaporating dust in their interiors. This leads to a significant drop in opacity and an associated increase in the surface temperatures reaching several hundred Kelvin or even higher.

In Fig. 5, we present surface density images, zoomed in on the four fragments in model 1 with mid-plane temperatures $T_{\text{mp},c}$ exceeding 10^3 K. The black curves outline the fragments as identified by our tracking mechanism. The yellow circles mark the Hill radii for each fragment defined as

$$R_{\text{H}} = r_{\text{f}} \left(\frac{1}{3} \frac{M_{\text{f}}}{M_{*} + M_{\text{f}}} \right)^{1/3}, \quad (27)$$

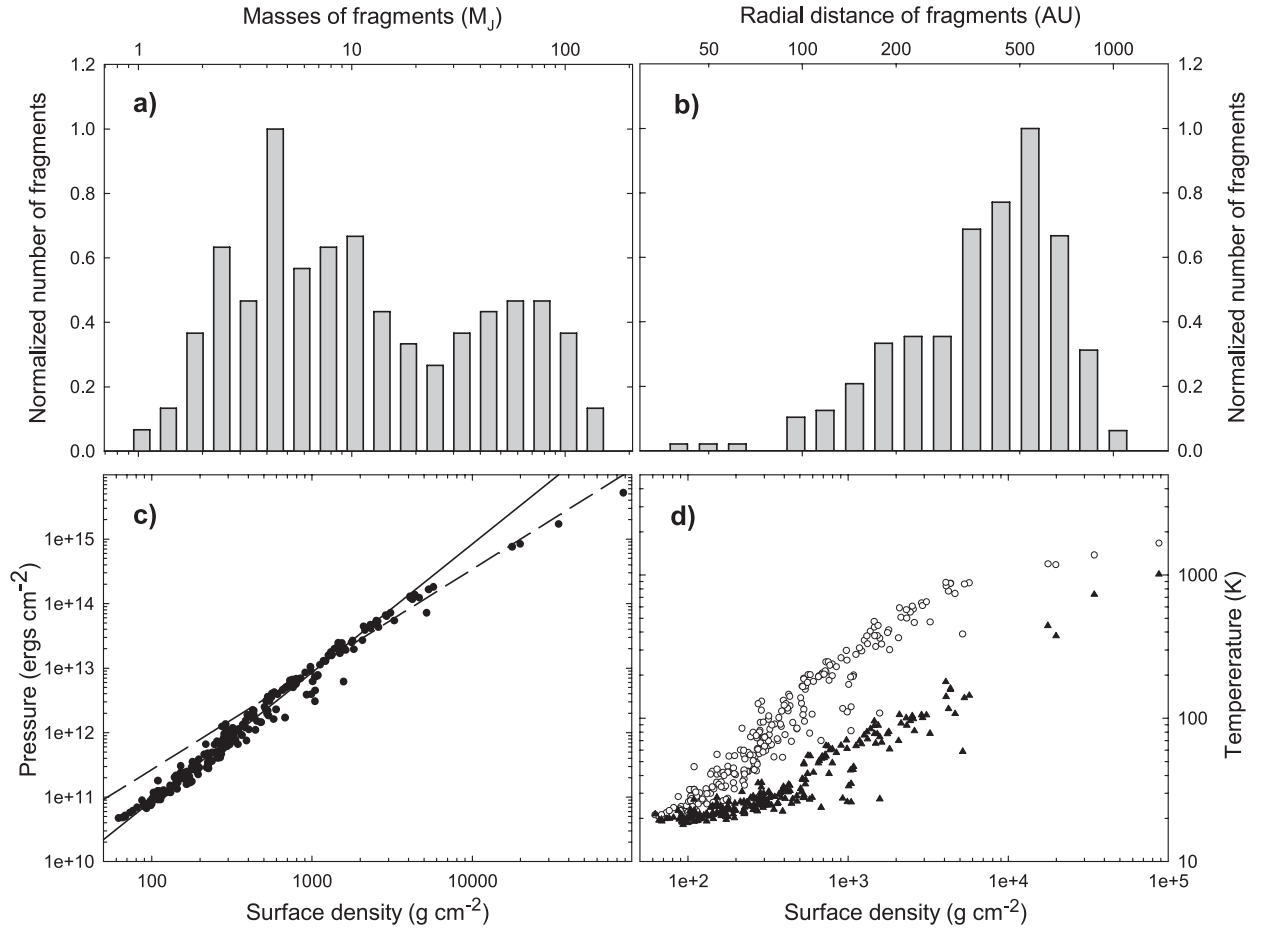


Figure 4. Properties of fragments in model 1. Panel (a): normalized distribution function of masses of the fragments. Panel (b): normalized distribution function of radial positions of the fragments. Panel (c): vertically integrated gas pressure \mathcal{P}_c versus gas surface density Σ_c at the geometrical centre of the fragments. Panel (d): mid-plane temperatures $T_{\text{mp},c}$ (open circles) and surface temperatures $T_{\text{surf},c}$ (filled triangles) versus Σ_c at the centres of the fragments. The lines are the best least-squares fits to the data with $\Sigma_c < 10^3 \text{ g cm}^{-2}$ (solid line) and $\Sigma_c \geq 10^3 \text{ g cm}^{-2}$ (dashed line). The corresponding exponents are 2.0 and 1.5.

where r_f is the radial distance from the protostar to the fragment and M_f is the mass of the fragment confined within the black curve.

The properties of the four fragments are listed in Table 1. In particular, the columns from left to right present: (1) the number of the fragment, (2) the time at which the fragment is seen in the disc, (3) the mass of the fragment, (4) the mass confined within the Hill radius M_H , (5) the radial distance to the fragment, (6) the radius of the fragment R_f , (7) the Hill radius of the fragment, (8) the mid-plane temperature of the fragment, (9) the surface temperature of the fragment, (10) the surface density of the fragment and (11) the optical depth to the mid-plane τ . The last four quantities were calculated at the geometrical centres of the fragments. The radius of the fragment is found by calculating the surface area occupied by the fragment and assuming that the fragment has a circular form.

Evidently, all four fragments with $T_{\text{mp},c} \geq 10^3 \text{ K}$ are in the brown-dwarf-mass regime. In fact, we do not find any planetary-mass objects with mid-plane temperatures exceeding 400 K. Fragments that are closer to the protostar are generally smaller in size, probably due to tidal stripping. We do not see any significant correlation between $T_{\text{mp},c}$ and r_f for the entire sample of the fragments in model 1, implying that hot fragments may be found at any distances and not only at $r < 170 \text{ au}$ as in the current model. The last column in Table 1 illustrates a strong dependence of opacity on mid-plane temperature. For fragment F2 with $T_{\text{mp},c} = 1180 \text{ K}$ the opacity is

$\tau = 256$, while for fragment F1 with $T_{\text{mp},c} = 1660 \text{ K}$ the opacity drops to $\tau = 18.5$. This drop is caused by evaporation of dust grains and results in a significant increase in the surface temperature, exceeding 10^3 K for the hottest fragment. A more detailed study of the properties of the fragments will be presented in a follow-up paper.

5 MODEL SPECTRAL ENERGY DISTRIBUTIONS

In this section, we calculate model SEDs using the method laid out in Section 2.3. Our object is split into three constituent parts: the protostar, the inner disc and the outer dynamic disc (plus envelope if present). Fig. 6 shows the azimuthally averaged gas surface density ($\bar{\Sigma}$; left-hand panels) and surface temperature (\bar{T}_{surf} ; right-hand panels) profiles in model 1 at the four times when the most massive fragments are present in the disc. The inner and outer discs are joined together at $r_j = 20 \text{ au}$. By assumption, the radial profiles of $\bar{\Sigma}$ and \bar{T}_{surf} in the inner disc are axisymmetric, but they are not in the outer disc – the averaging procedure could not smooth out azimuthal inhomogeneities caused by spiral arms and fragments. The most massive fragments manifest themselves by clear-cut peaks in $\bar{\Sigma}$ at $r \approx 50 \text{ au}$ ($t = 0.09 \text{ Myr}$), $r \approx 110 \text{ au}$ ($t = 0.1 \text{ Myr}$) and

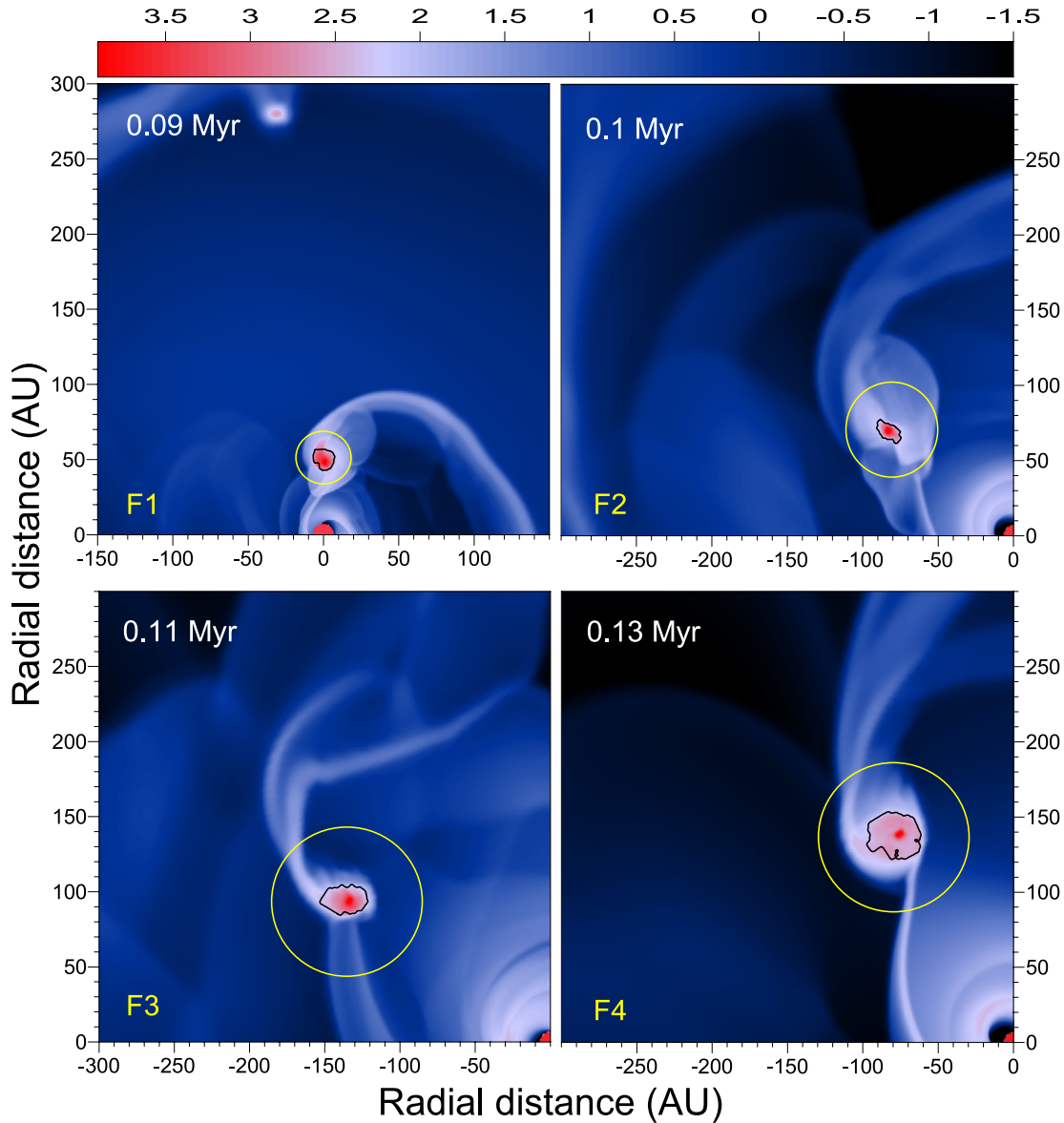


Figure 5. Zoomed-in surface density images of four fragments in model 1 characterized by mid-plane temperature exceeding 10^3 K. The black curves outline the fragments as found by our fragment tracking algorithm. Yellow circles mark the corresponding Hill radii. Time elapsed since the formation of the central protostar is indicated in each panel. The scale bar is in g cm^{-2} .

Table 1. Characteristics of the fragments.

Fragment	Time (Myr)	M_f (M_{Jup})	M_H (M_{Jup})	r_f (au)	R_f (au)	R_H (au)	T_{mp} (K)	T_{surf} (K)	Σ (g cm^{-2})	τ
F1	0.09	55	68	48	8	17	1660	1020	8.7×10^4	18.5
F2	0.1	32	47	108	9	32	1180	380	2.0×10^4	256
F3	0.11	52	63	163	16	56	1190	445	1.8×10^4	139
F4	0.13	64	84	158	21	55	1375	740	3.5×10^4	32

M_f – the mass of the fragment, M_H – the mass confined within the Hill radius, r_f – the radial distance to the fragment, R_f – the radius of the fragment, R_H – the Hill radius of the fragment, T_{mp} – the mid-plane temperature of the fragment, T_{surf} – the surface temperature of the fragment, Σ – the surface density of the fragment and τ – the optical depth to the mid-plane of the fragment.

$r \approx 160$ au ($t = 0.11$ Myr and $t = 0.13$ Myr). There are also a number of secondary peaks corresponding to less massive fragments. \bar{T}_{surf} exceeds 10^3 K near the dust sublimation radius ($R_{\text{in}} \approx 0.1\text{--}0.15$ au, depending on the total luminosity) and declines gradually to 10 K

in the envelope. There are also local maxima seen in \bar{T}_{surf} at the positions of the fragments but their magnitudes are considerably smaller than those of Σ due to substantial input of isotropic stellar irradiation into the thermal balance of the outer disc.

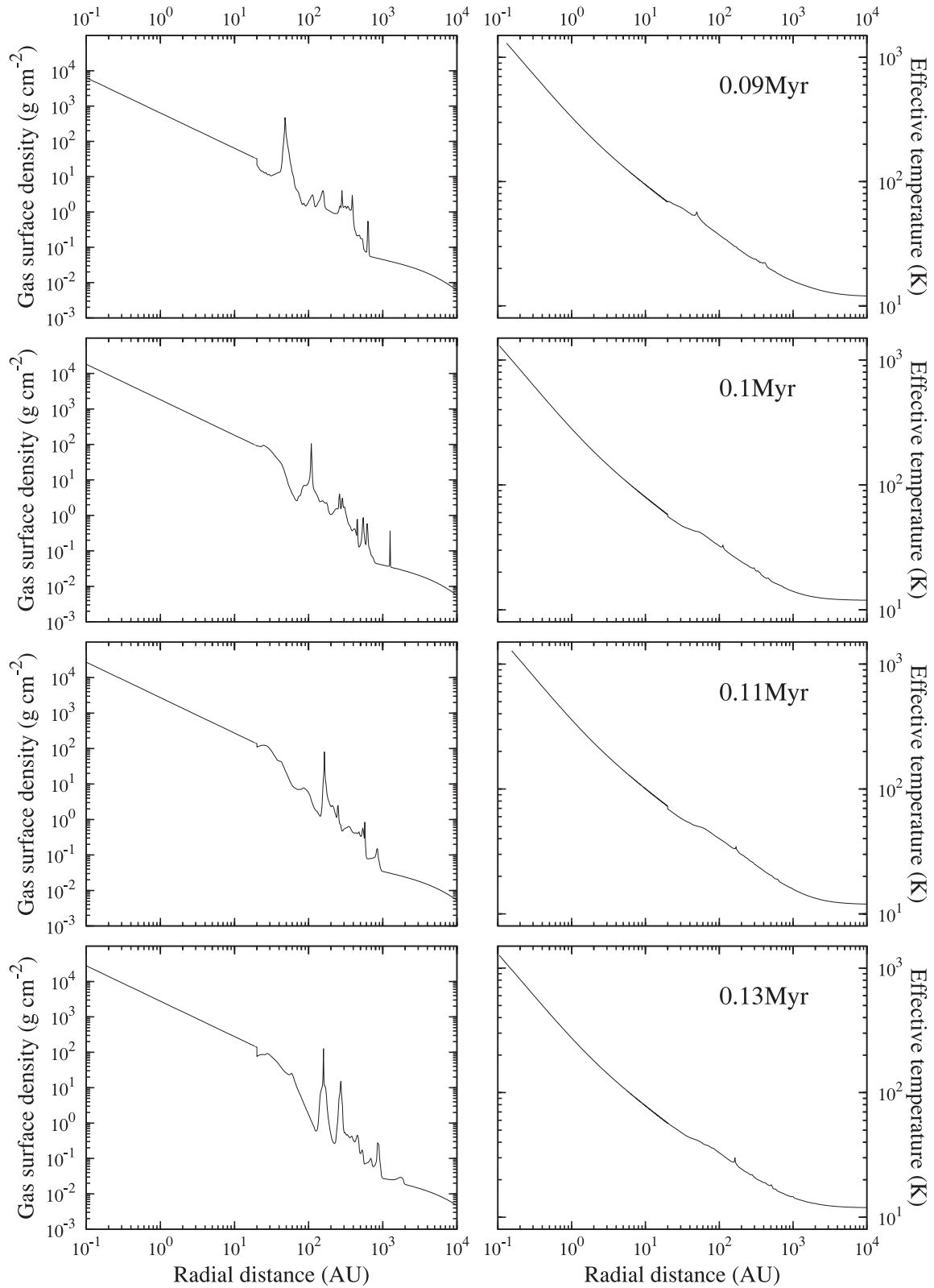


Figure 6. Azimuthally averaged gas surface density (left-hand column) and effective temperature (right-hand column) distributions in our model at four times after the formation of the central protostar.

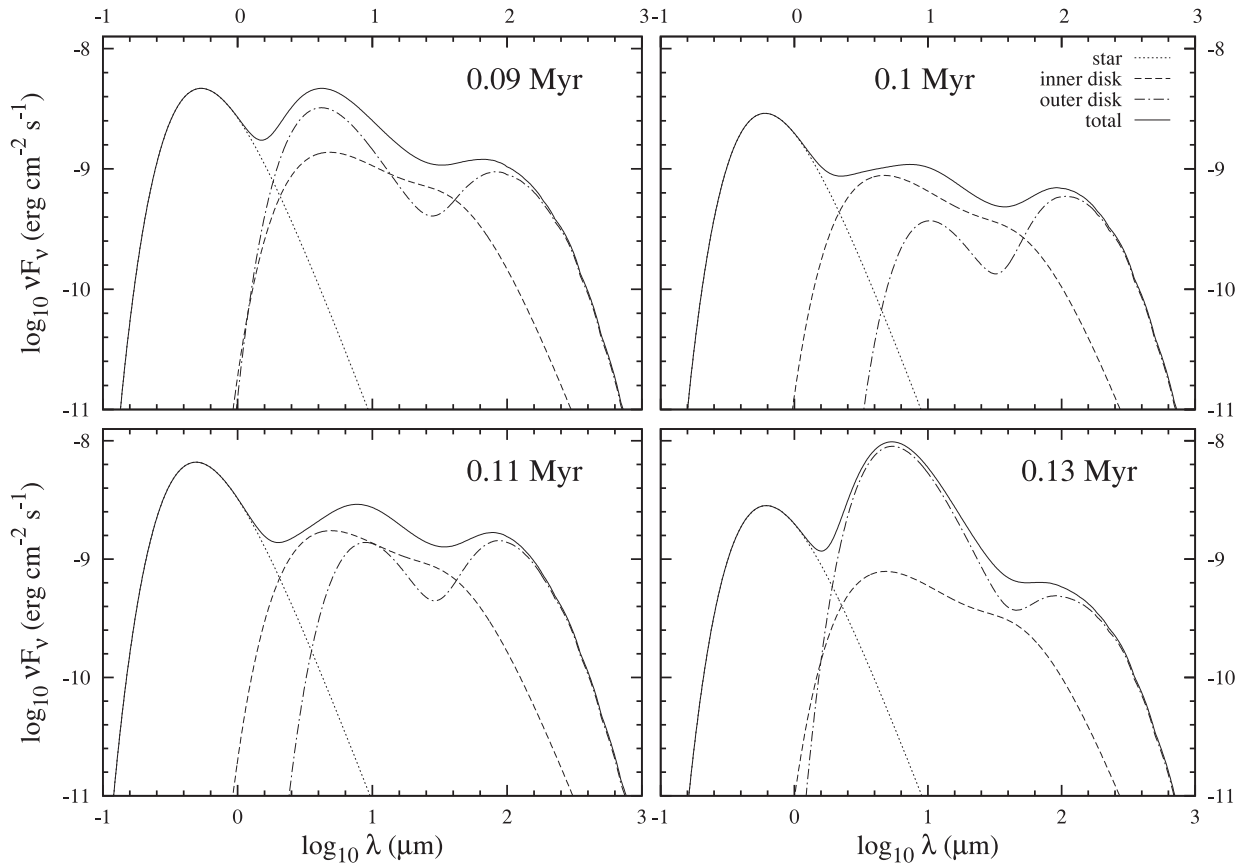


Figure 7. Model SEDs in model 1 at four times after the formation of the central star. Individual contributions from the central protostar, inner disc and outer disc are shown by the dotted, dashed and dash–dotted lines, respectively. The solid line is the total flux.

Fig. 7 presents the resulting SEDs in model 1. In particular, individual contributions from the central protostar, inner disc and outer disc are shown by the dotted, dashed and dash–dotted lines, respectively. The solid line is the total flux νF_ν in units of $\text{erg cm}^{-2} \text{s}^{-1}$. The distance to our model object is taken to be 250 pc.

A visual inspection of Fig. 7 reveals that hot and massive fragments may contribute significantly to the total radiative flux. In particular, the contribution from the outer disc has a characteristic double-peaked structure, with the peak at shorter wavelengths produced by the most massive and hot fragments in the disc. The input from the fragments to the total flux may sometimes exceed that from the inner disc (dotted lines). As a result, the total flux may have a characteristic secondary peak at $\lambda \approx 5 \mu\text{m}$ comparable in magnitude to that produced by the protostar at $\lambda \approx 0.5 \mu\text{m}$. This feature is clearly visible at $t = 0.09 \text{ Myr}$ and $t = 0.13 \text{ Myr}$ when fragments F1 and F4 have mid-plane temperatures exceeding 1300 K. At these high temperatures dust grains start to evaporate, which leads to a sharp drop in opacity and the corresponding increase in the surface temperature. As Table 1 demonstrates, T_{surf} exceeds 700 K for fragment F1 at $t = 0.09 \text{ Myr}$ and fragment F4 at $t = 0.13 \text{ Myr}$. In comparison, fragment F2 at $t = 0.1 \text{ Myr}$ and fragment F3 at $t = 0.11 \text{ Myr}$ are characterized by mid-plane temperatures that are below the dust sublimation temperature of $\approx 1200 \text{ K}$. Their surface temperatures are below 450 K, their input to the total SEDs is comparable to that of the inner disc, and the secondary peak at $\lambda \approx 5 \mu\text{m}$ is less evident.

To highlight the predicted SED differences between fragmenting and non-fragmenting discs, we show in Fig. 8 the SEDs for the

non-fragmenting model 2. Four ages of the disc that approximately match those of model 1 in Fig. 7 were considered. Evidently, the SED of the outer disc (dash–dotted lines) has a single-peaked form with a maximum at $\approx 100 \mu\text{m}$. The outer disc contributes a negligible fraction to the total flux at $\lambda \approx 5 \mu\text{m}$, unlike the fragmenting model 1 showing a secondary peak at this wavelength. Nevertheless, there is a small feature at $\lambda \approx 5 \mu\text{m}$, which is not related to the outer disc and is likely caused by simplistic modelling of the inner disc due to the limitations of our numerical hydrodynamics scheme.

Several recent studies have presented observations and models of the SEDs of relatively large samples of embedded protostars and T Tauri stars. For example, Robitaille et al. (2007); Furlan et al. (2008, 2011) all presented such data and models for targets in Taurus. No triple-peaked SEDs like those seen at 0.09, 0.11 or 0.13 Myr in Fig. 7 are seen in these studies, suggesting that such massive fragments are not present most of the time. Indeed, such massive fragments should only last at most a few thousand years before migrating on to the star, approximately 1 per cent of the embedded stage duration of a few $\times 10^5 \text{ yr}$ (Evans et al. 2009; Vorobyov 2010; Dunham & Vorobyov 2012). Even if several fragments form and migrate inward during the embedded stage, the total time when such features are observable is likely less than 10 per cent, especially considering the necessity of favourable viewing geometry. Thus, it is not surprising that no triple-peaked SEDs like those shown in Fig. 7 have been detected given that studies like those listed above generally only contain a few dozen embedded sources. Should such a feature be detected in the future, it would be a strong indication of a massive disc fragment.

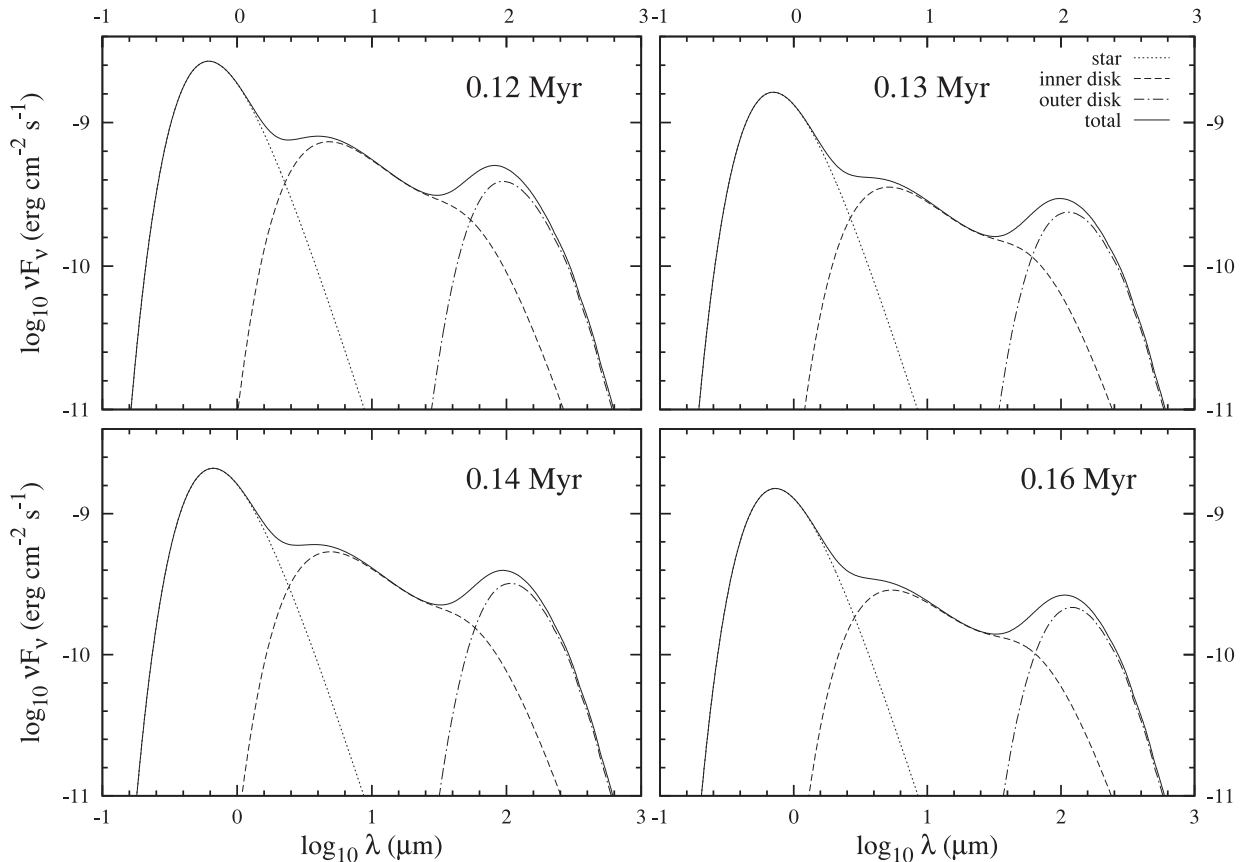


Figure 8. Same as for Fig. 7 only for model 2.

6 SYNTHETIC ALMA IMAGES OF FRAGMENTING DISCS

In the previous section, we have shown that the most massive and hot fragments can leave characteristic signatures in the SEDs of young protostellar discs observed through outflow cavities. In this section, we explore the possibility of detecting the fragments directly using interferometric observations with the ALMA.

To investigate this possibility, we first generated synthetic images following the same procedures described above for producing SEDs except, instead of summing over all spatial positions, we calculated the intensity in each pixel in a Cartesian coordinate grid with a grid spacing of 5 au (0.025 arcsec at the assumed distance of 250 pc). We then used the *simobserve* and *simanalyze* tasks in the *CASA* software package³ to simulate 1 h of ALMA integration at both 230 and 345 GHz, where these particular frequencies were chosen as the best compromise between resolution, sensitivity and availability of suitable weather for such observations. The antenna configuration most closely corresponding to a synthesized beam of 0.1 arcsec (25 au at the assumed distance of 250 pc) was selected for each frequency, since such a resolution is the minimum necessary to adequately resolve the disc into its individual fragments. With these simulated observing parameters, the 3σ detection limits of the ALMA images are approximately 0.2 mJy beam⁻¹ at 230 GHz and 0.8 mJy beam⁻¹ at 345 GHz.

³ *CASA*, the Combined Astronomy Software Applications, is the ALMA data reduction and analysis package; see <http://casa.nrao.edu/> for details.

Fig. 9 presents the gas surface densities in model 1 (top row, log g cm⁻²) and simulated ALMA images at 230 (middle row) and 345 GHz (bottom row), both in log mJy beam⁻¹. The actual input images to the ALMA data simulator are flux density images as described above, not surface density images, but these input images are qualitatively very similar to the surface density images shown in the top row. Both 230 and 345 GHz are sufficiently in the Rayleigh–Jeans limit such that the emission is only linearly dependent on temperature and thus primarily traces surface density, which varies over a much larger range than the temperature. The time elapsed since the formation of the protostar is indicated in the top row and the beam size is plotted in the right-bottom corner of each image. The black arrows in the top row point to the most massive fragments shown in Fig. 5 and described in detail in Table 1.

Evidently, some of the fragments can be confidently detected with ALMA using just one hour of integration time. For instance, fragments F2–F4 at a radial distance of about 100–150 au are clearly seen in the image and their peak fluxes are well above the detection limits of 0.2 (0.8) mJy beam⁻¹ at 230 (345) GHz. On the other hand, fragments that are located at radial distances $r \lesssim 50$ au start to merge with the central flux peak and are more difficult to detect with our chosen resolution of 0.1 arcsec. For instance, fragment F1 (at $r_f = 48$ au) is beginning to merge with the central peak. This is not surprising considering that the corresponding linear resolution is 25 au for the adopted distance of 250 pc. However, we note that, once ALMA begins full-science array operations, angular resolutions better than 0.1 arcsec will be achievable, enabling even very closely spaced fragments to be separated (although longer

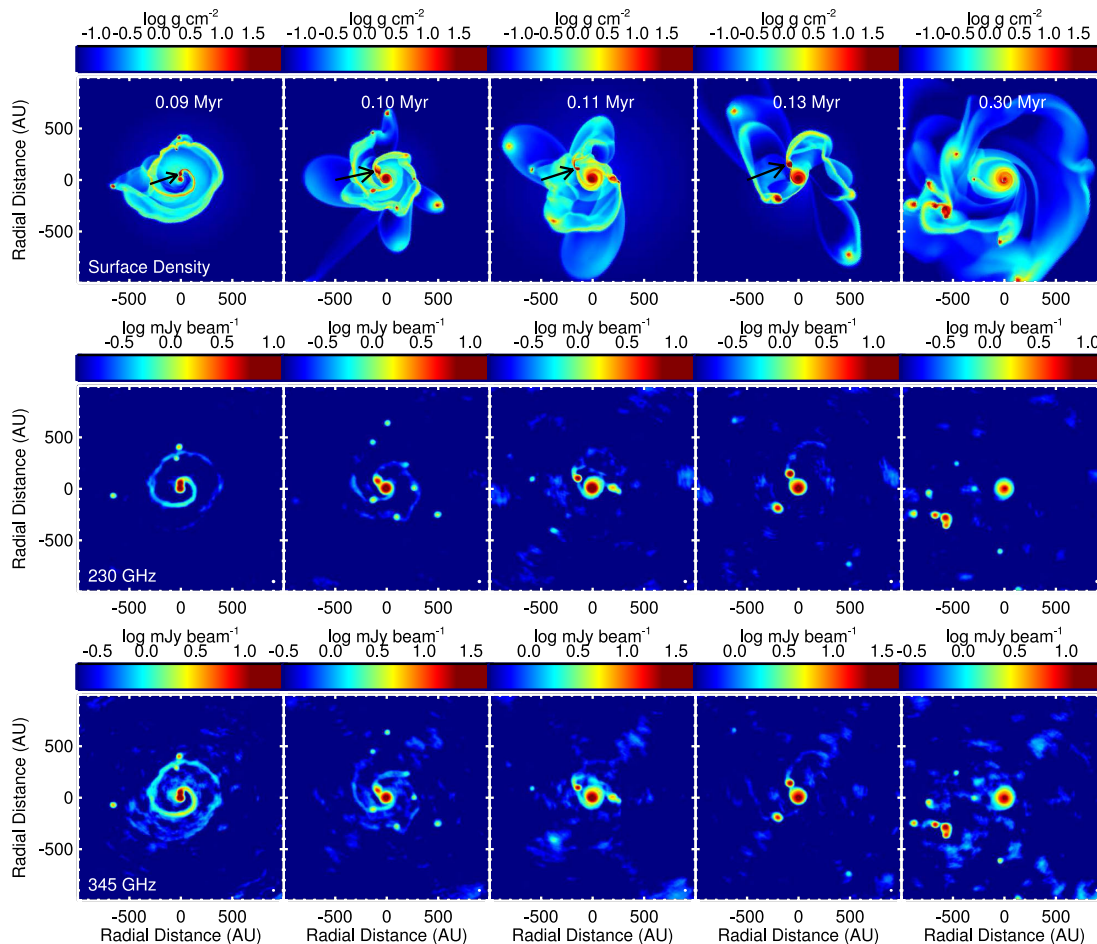


Figure 9. Top row: gas surface density maps of the inner 2000×2000 au at five times since the formation of the central protostar. The black arrows point to the most massive fragments F1–F4. Middle and bottom rows: the corresponding synthetic ALMA images at 230 (middle) and 345 GHz (bottom). The beam size is indicated in white in the bottom right of each panel. The images are displayed on a logarithmic scaling to capture the full dynamic range of the central density peaks, surrounding fragments and spiral disc structure.

integration times than the 1 h considered here will be required to reach comparable sensitivities). We also note that the spiral structure is clearly evident in some images.

While the log scaling is useful to emphasize the spiral structure, it unfortunately also enhances the noise. Images with a linear scaling chosen to oversaturate the central density peaks highlight the weak, distant fragments, thus Fig. 10 presents such images for the same five times as in Fig. 9. The maximum image intensities exceed 20 mJy beam^{-1} but we have set the maximum of the image scales to values in the approximate range of $1\text{--}4 \text{ mJy beam}^{-1}$, thus oversaturating the images by factors of 3 or more. Evidently, this helps to resolve weaker and more distant fragments at the cost of losing fragments at $r_f \lesssim 100$ au, which are now almost completely merged with the central density peaks.

The yellow arrows in the middle and bottom rows of Fig. 10 indicate the least massive fragments that can be detected with ALMA. These fragments have masses of 3.5 , 2.5 , 2.0 , 3.5 and $1.5 M_{\text{Jup}}$ at $t = 0.09$, 0.10 , 0.11 , 0.13 and 0.3 Myr respectively. Obviously, ALMA can detect fragments down to the planetary-mass regime. The largest orbital distance of detectable fragments is about 850 au.

To illustrate the effect of reduced resolution, we generated the simulated ALMA images in model 1 at the same five times as in Fig. 10 but for a resolution of 0.5 arcsec. The corresponding images

are shown in Fig. 11 using the same oversaturated scaling as in Fig. 10. Evidently, fragments located at radial distances from the protostar $\lesssim 200$ au have completely merged with the central peak. In addition, close-separation fragments now appear as one fragment (e.g. at $t = 0.09$ and 0.3 Myr). Consequently, the total number of identifiable fragments has reduced considerably. We conclude that a resolution of 0.5 arcsec can be used for discs located at distances $\lesssim 150$ pc.

7 MODEL CAVEATS

Uncertainties in dust opacity. In this study, we used the frequency-integrated opacities of Bell & Lin (1994) to calculate the disc thermal balance and also the frequency-dependent opacities of Ossenkopf & Henning (1994) for the case of thin ices to calculate the flux. Uncertainties in the former may affect the calculated surface temperatures and hence the shape of the model SEDs. In particular, if dust grains evaporate at higher temperatures than assumed in Bell & Lin (1994), then the characteristic secondary peak at $\lambda \approx 5 \mu\text{m}$ (see Fig. 7) will be less pronounced. Numerical simulations with other opacity tables are needed to assess the effect.

On the other hand, uncertainties in the adopted frequency-dependent opacities have little effect on the resulting SEDs at

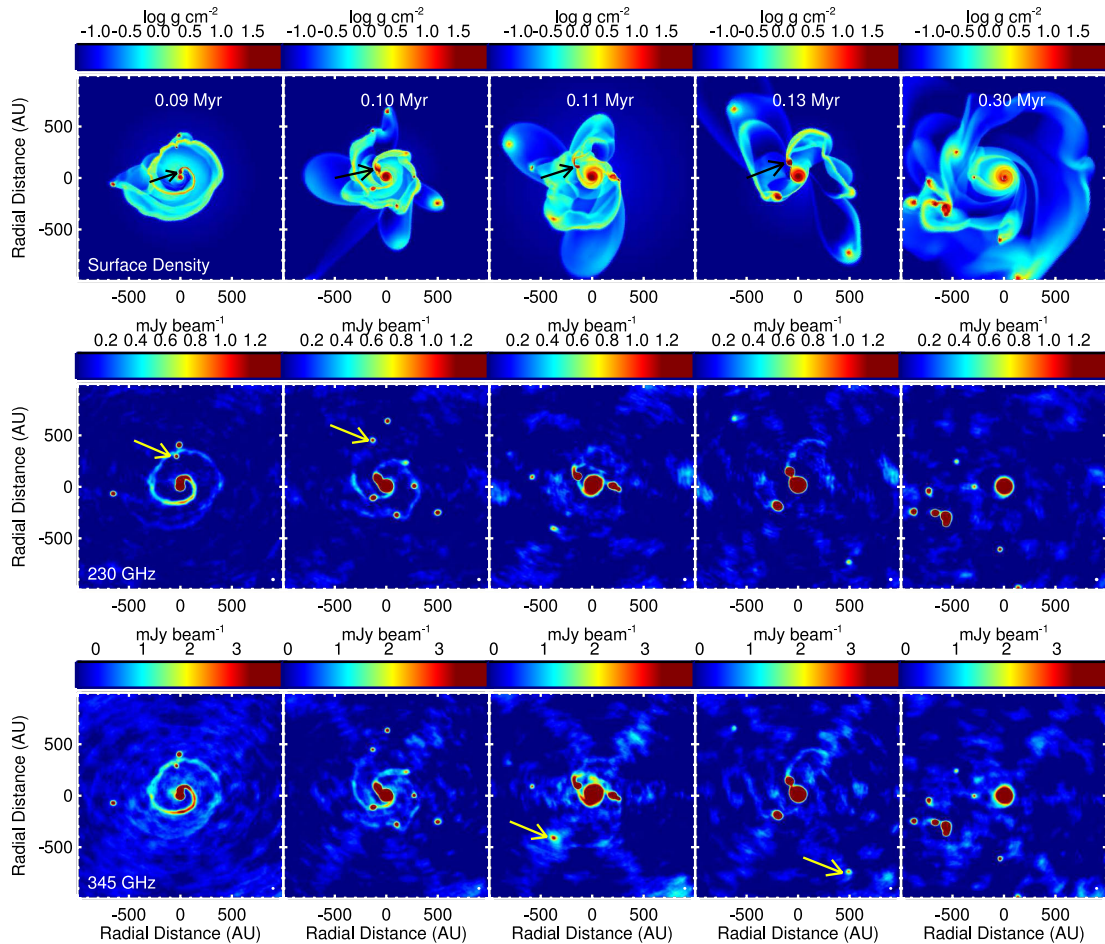


Figure 10. Same as Fig. 9, except the images are displayed on a linear scaling with the scale ranges chosen to emphasize the detections of the disc fragments above the disc spiral structure, at the expense of oversaturation of the central density peaks. The yellow arrows highlight the least massive fragments clearly detected in the images.

$\lambda \lesssim 100 \mu\text{m}$ due to high optical depth at these frequencies but may have some effect at longer wavelengths. To evaluate the magnitude of this effect, we calculated the SEDs with frequency-dependent opacities of Ossenkopf & Henning (1994) for the three cases of thin ice, thick ice and no ice. The resulting radiative fluxes at 230 and 345 GHz differ by a factor of 2 at most. Finally, we note that opacity is varying vertically, an effect that cannot be captured in our 2D approach.

Non-zero inclination. In this paper, we have constructed SEDs and synthetic ALMA images using a methodology of Chiang & Goldreich (1997) and assuming zero inclination, i.e. for objects viewed pole-on. Our method is best applied to a later evolutionary stage when most of the parental core has dissipated, thus neglecting possible photon scattering or reprocessing by the envelope. To better compare with other studies that present synthetic SEDs of protostars based on global simulation data (e.g. Kurosawa et al. 2004; Offner et al. 2012b), we ran simple Monte Carlo radiative transfer models of a star+disc system embedded within a core. For this purpose, we used the dust radiative transfer package `RADMC` (Dullemond & Dominik 2004). The SEDs of the net star+disc system shown in Fig. 7 were adopted as the input SEDs. A flattened, rotating density structure following the Terebey, Shu & Cassen (1984) solution, with an outflow cavity with an opening semi-angle of 30° was assumed for the core, and the adopted dust opacities

were taken from OH5 (Ossenkopf & Henning 1994), modified to include both absorption and isotropic scattering opacities (see Dunham et al. (2010); Dunham & Vorobyov (2012) for details). The results are shown in Fig. 12. Reprocessing by the dust in the core becomes significant for high inclinations but is negligible for inclinations smaller than the semi-opening angle of the outflow cavity, and the excess emission at long wavelengths from dust in the core does not hinder the ability to detect the triple-peaked SED indicative of massive disc fragments. Furthermore, our models do not suggest that scattering will significantly affect the observed mid-infrared SED to the point of hindering the detection of massive disc fragments, but we acknowledge that our models likely underestimate this effect since we are unable to properly treat scattering as anisotropic.

We also note that for large values of γ_{incl} , the position angle of the fragment in the disc becomes important. The fragment may be enhanced/suppressed if it is located on the farthest/nearest side of the disc with respect to the observer.

Background irradiation. The temperature of background irradiation is set to 10 K in the present study. A higher temperature of background irradiation makes the overall disc warmer. The net result is that the Jeans length is increasing and the fragments become more massive, which may facilitate their detection. At the same time, the disc becomes less prone to fragmentation and, as numerical

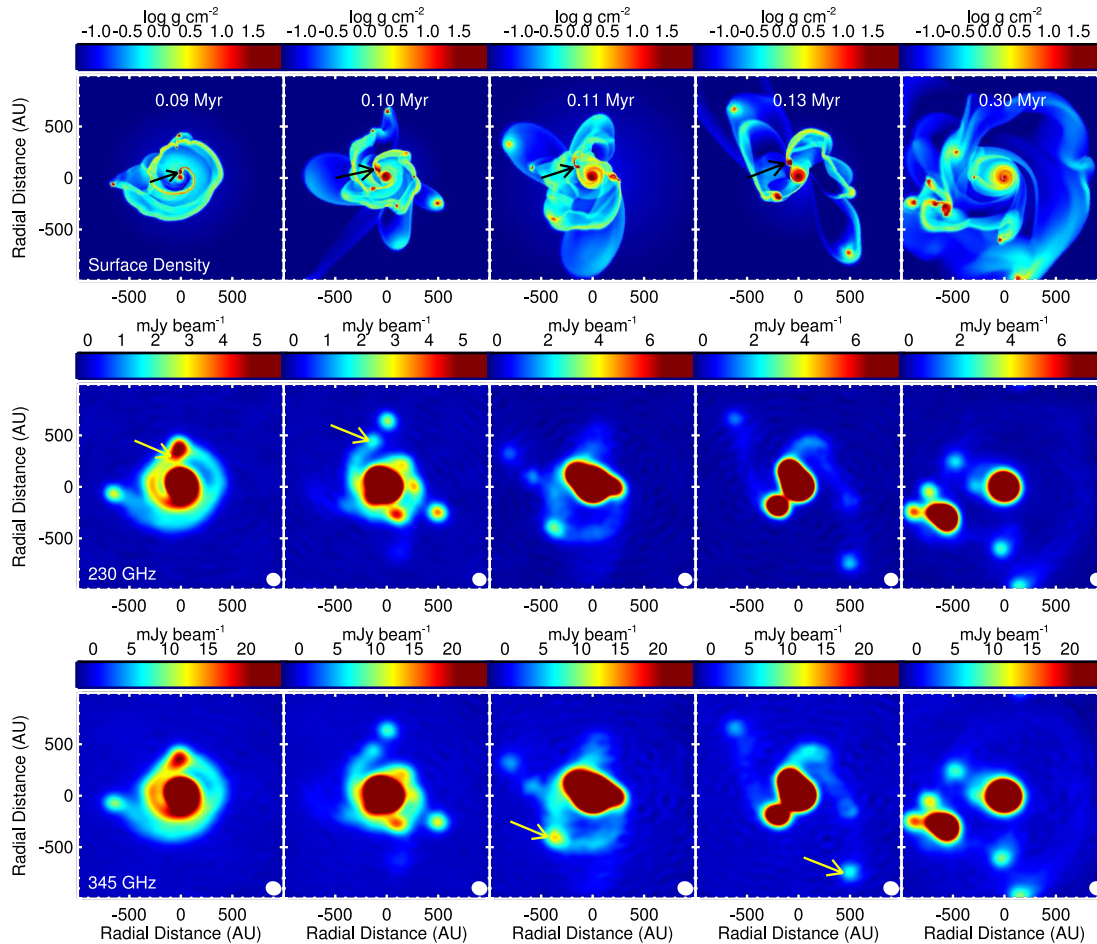


Figure 11. Same as Fig. 10, except for a reduced resolution of 0.5 arcsec.

hydrodynamics simulations of Vorobyov & Basu (2006) demonstrated, fragmentation ceases at $T_{\text{bg}} \gtrsim 50$ K.

Dust sublimation temperature. In this study, we adopted a dust sublimation temperature of 1500 K to calculate the sublimation radius in equation (18). We found that varying T_{ds} in the 1000–2000 K limits influences our results insignificantly. In particular, for $T_{\text{ds}} = 1000$ K the maximum decrease in the flux by 18 per cent (with respect to that calculated for $T_{\text{ds}} = 1500$ K) occurs at $\lambda = 1.2$ μm , while for $T_{\text{ds}} = 2000$ K, the corresponding increase in the flux by 46 per cent occurs at $\lambda = 1.75$ μm .

Numerical resolution. The Truelove criterion (Truelove et al. 1998) states that the Jeans length has to be resolved by at least four grid zones in order to correctly study disc fragmentation. The Jeans length R_J of a thin, fragmenting disc can be expressed as (Vorobyov 2013)

$$R_J = \frac{\langle v^2 \rangle}{\pi G \Sigma_d}, \quad (28)$$

where $\langle v^2 \rangle = 2\mathcal{R}T_d/\mu$ is the velocity dispersion of a thin disc, \mathcal{R} is the universal gas constant, T_d is the gas mid-plane temperature in the fragment and Σ_d is the gas surface density at the fragment–disc interface. Fragments usually condense out of densest sections of spiral arms. The typical surface densities and temperatures in spiral arms do not exceed 100 g cm^{-2} and 100 K (Vorobyov 2011). Adopting these values for Σ_d and T_d , the corresponding Jeans length is $R_J \approx 20$ au.

In our model, the radial and azimuthal grid resolution at $r = 100$ au is ≈ 1.0 au and the Jeans length is resolved by roughly 20 grid zones in each coordinate direction, sufficient to fulfil the Truelove criterion. The resolution deteriorates with increasing distance from the central star and the Truelove criterion is expected to break at $r \gtrsim 500$ au, where the spatial resolution on our logarithmically spaced grid starts to exceed 5.0 au and the Jeans length is resolved by less than four grid zones in each coordinate direction. However, fragmentation takes place mostly at radial distances from a few tens to a few hundreds au. Fragments that are seen in our model at larger distances are most likely scattered from the inner disc regions due to gravitational interaction with other fragments.⁴ Higher resolution simulations are planned for the near future to confirm our results.

Contraction time-scale. The grid resolution imposes limitations on the minimum size of a fragment of the order of 1.0 au. This means that we cannot follow the contraction of fragments to planetary- or sub-stellar-sized objects and hence we may overestimate the feasibility of using (sub-)millimeter observations to detect the presence of fragments (fully formed planetary or sub-stellar objects will become essentially invisible for the ALMA). The contraction time-scale of a fragment to a fully formed object strongly depends on its initial mass. A one-Jupiter-mass fragment will contract for about a

⁴ In the most extreme cases, some of the fragments may even be ejected from the disc into the intracluster medium (Basu & Vorobyov 2012).

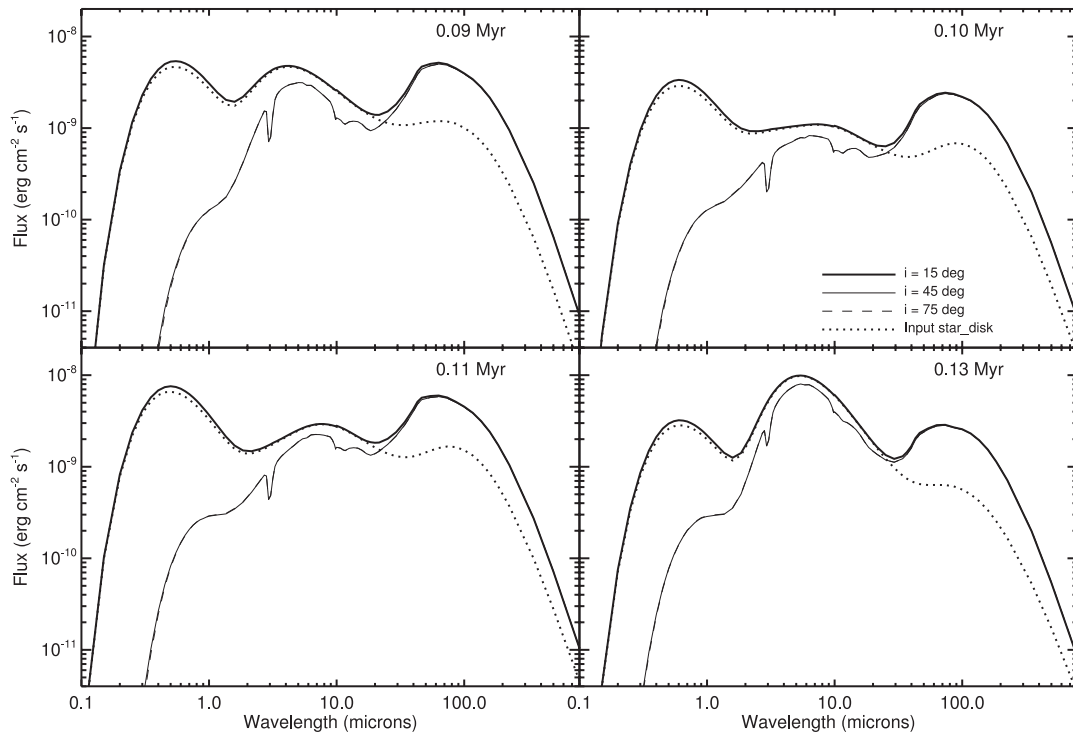


Figure 12. SEDs for model 1 at the same four times shown in Fig. 7 obtained from Monte Carlo radiative transfer models of the net star+disc system embedded in a core (see the text for details). In each panel, the model SEDs at three different inclinations are plotted, along with the star+disc SED given as an input to the models. Note that the $i = 45^\circ$ and $i = 75^\circ$ curves nearly merge due to an almost spherical core shape.

few $\times 10^5$ yr, while a ten times more massive fragment will collapse in $< 10^4$ yr (Tohline 2002; Boley et al. 2010; Helled & Bodenheimer 2010). These time-scales are comparable to or shorter than the time-scale between successive bursts of disc fragmentation, $(7 - 12) \times 10^4$ yr (see Fig. 3), which periodically regenerate the population of fragments in the disc. This implies that the probability of detecting fragments increases with decreasing mass of the fragment and it may be quite difficult to ‘catch’ a massive proto-brown dwarf in the process of formation.

The effect of environment and magnetic fields. In the present study, we considered the formation of protostellar discs from isolated non-magnetized cores. Numerical hydrodynamics simulations of star cluster formation indicate that radiation feedback from young stars and tidal torques can significantly reduce the disc propensity to fragment (e.g. Bate 2009; Offner et al. 2009). Our model takes into account disc irradiation by the central protostar and background blackbody radiation with a temperature appropriate for isolated cores, $T_{\text{bg}} = 10$ K, but does not include tidal stripping of the disc. Our results are thus more applicable to isolated star formation in a quiescent environment. In addition, magnetic braking can significantly reduce the disc mass and size in the early Class 0 phase (sometimes even preventing disc formation), also diminishing the disc propensity to fragment (Li, Krasnopolsky & Shang 2011; Joos, Hennebelle & Ciardi 2012). The effect of magnetic fields can thus reduce the likelihood of detecting the fragments.

8 CONCLUSIONS

Using numerical hydrodynamics simulations in the thin-disc limit, we studied the formation and evolution of a gravitationally unsta-

ble protostellar disc with a particular emphasis on the properties of fragments formed via disc gravitational fragmentation. We constructed the SED of our model object viewed pole-on, including the flux from the central protostar, the inner unresolved disc and the outer disc plus envelope system. We also used the ALMA software and generated synthetic images of our model disc at two frequencies best fitted for the fragment detection. We found the following.

(i) The disc experiences multiple episodes of fragmentation in the embedded phase of star formation, with the time between recurrent fragmentation episodes approximately equal to the characteristic time of mass infall on to the disc M_{d}/\dot{M} . The fragments can therefore be present in the disc for a significantly longer time period than expected for just one burst of disc fragmentation in numerical hydrodynamics simulations without disc infall.

(ii) The mass distribution function of the fragments have two maxima, one in the planetary-mass regime around $5M_{\text{Jup}}$ and the other in the upper brown-dwarf-mass regime around $60-70M_{\text{Jup}}$. The whole mass spectrum extends from about a Jupiter mass to very-low-mass stars. The radial positions of the fragments have a maximum at 400–500 au, with a dearth of fragments at $r < 100$ au caused presumably by fast inward migration at these distances.

(iii) The majority of the fragments are characterized by surface temperatures $T_{\text{surf}} < 100$ K due to high optical depths. Occasionally, some fragments may attain temperatures in their interiors sufficient to evaporate dust grains. Such fragments are characterized by much higher surface densities and can leave characteristic peaks at $\approx 5 \mu\text{m}$ in the SEDs of young protostellar discs viewed through the outflow cavity. These features are comparable in magnitude to the flux from

the central star and can potentially be used to infer the presence of massive ($\gtrsim 50M_{\text{Jup}}$) and hot fragments in protostellar discs. Non-fragmenting discs lack notable peaks around $5\ \mu\text{m}$.

(iv) The majority of the fragments in our model can be detected with ALMA using just one hour of integration time and a resolution of $0.1\ \text{arcsec}$, provided that the disc is favourably inclined. The detection limit on the mass of the fragments is $1.5M_{\text{Jup}}$ at a distance of $250\ \text{pc}$. It is advisable to use a log scaling to resolve the spiral structure and fragments at small distances $\lesssim 100\ \text{au}$, while the over-saturated linear scaling is best fitted to detect distant and low-mass fragments. The likelihood of detecting the fragments reduces significantly for a lower resolution of $0.5\ \text{arcsec}$ but can probably be used for objects located at distances of the order of $150\ \text{pc}$ and less.

In this paper, we have explored only one fragmenting model. More numerical simulations exploring a wide parameter space of initial core masses, the effects of different inclinations and environments are further needed.

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APPENDIX A: TESTING THE SED ALGORITHM

To test the SED algorithm described in Section 2.3, we tried to reproduce the model of Chiang & Goldreich (1997) for a flared disc heated by stellar irradiation. In this model, the disc surface temperature is calculated as

$$T_{\text{surf}} = \left(\frac{\alpha}{2}\right)^{1/4} \left(\frac{R_*}{r}\right)^{1/2} T_*, \quad (\text{A1})$$

where the grazing angle is $\alpha = 0.005r_{\text{au}}^{-1} + 0.05r_{\text{au}}^{2/7}$ and r_{au} is the radial distance in au. We used this temperature distribution and applied the inner and outer disc cutoff radii $0.07\ \text{au}$ and $200\ \text{au}$, respectively.⁵ The star was modelled as a spherical blackbody with temperature $T_* = 4000\ \text{K}$ and radius $R_* = 2.5\ R_{\odot}$, as in Chiang & Goldreich (1997).

⁵ We note that Chiang & Goldreich (1997) used a value of $270\ \text{au}$ for the outer disc radius.

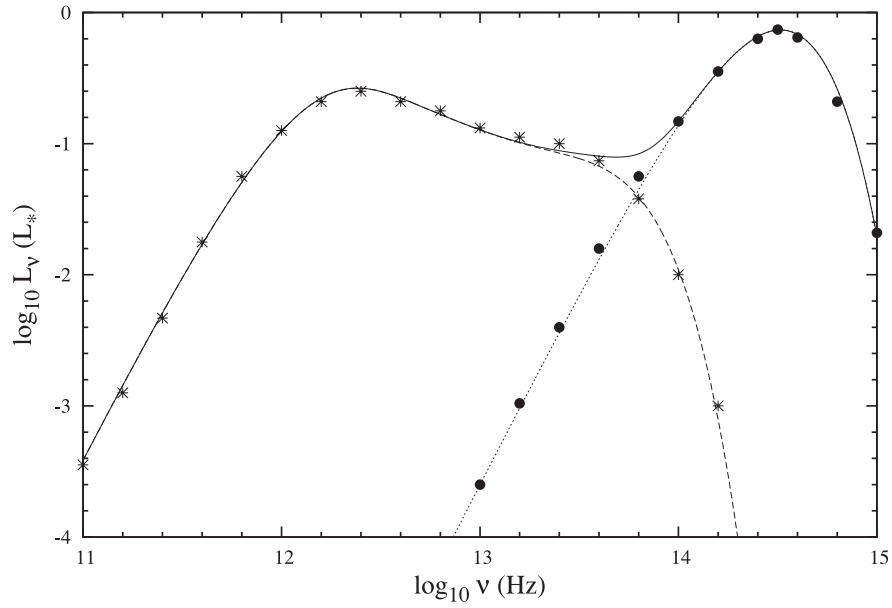


Figure A1. Comparison of our test SED with that of Chiang & Goldreich (1997). Individual contributions to the test SED from the central star and the disc are shown with dotted and dashed lines, respectively; the solid line is the total flux. The SED of Chiang & Goldreich are plotted with asterisks (disc input) and filled circles (stellar input).

The comparison of the resulting SED with that presented in Chiang & Goldreich (1997) is shown in Fig. A1. Our SED is shown by the solid line, with the individual contributions of the disc and the star plotted by dashed and dotted lines, respectively. The SED of Chiang & Goldreich (1997) is indicated with filled circles (stellar

input) and asterisks (disc input), respectively. Fig. A1 indicates that our algorithm can reasonably well reproduce the results of Chiang & Goldreich (1997).

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