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Can Inhomogeneous Primordial Magnetic Field Affect Nucleosynthesis in the Early Universe?

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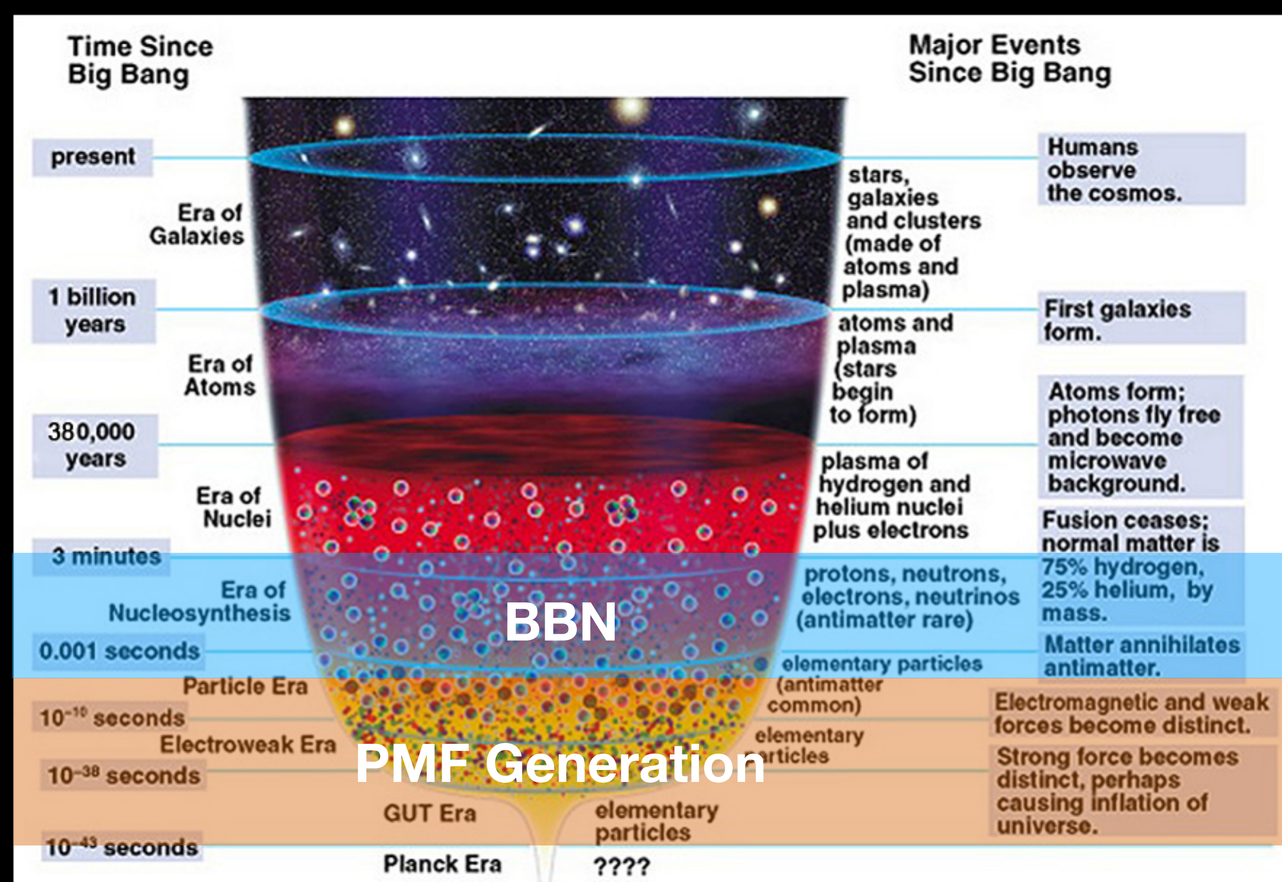

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Primordial Magnetic Field Impacts

Magnetic field and its impact on thermodynamics

Magnetic momentum of particles $\mu_B \propto \frac{e\hbar}{2m}$

- Mass of electron $m_e = 0.511\ \mathrm{MeV}$

$\wedge$
$\wedge$

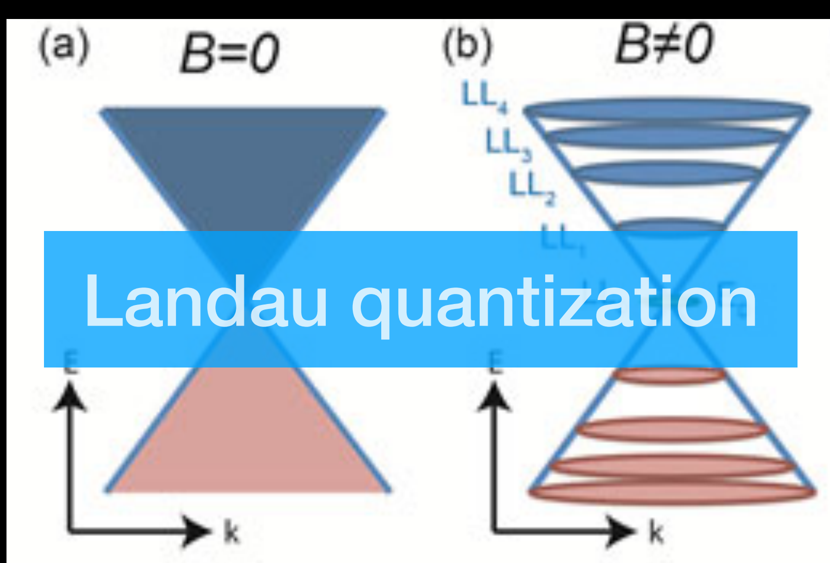
- Mass of proton $m_p = 938\ \mathrm{MeV}$

→ Main effects are on electron-positron

$B \neq 0$

Electron (Positron) energy $E_0^2 = p^2 + m_e^2 + 2eBn$ $(c = \hbar = 1)$

$$\sum_{n=0}^{\infty} (2 - \delta_{n0}) \frac{\mathrm{d}p_z}{2\pi} \frac{eB}{2\pi} f_{FD}(E_B, T)$$



Number density $$n_e = \frac{eB}{(2\pi)^2} \sum_{n=0}^{\infty} (2 - \delta_{n0}) \int f_{ED}(E_B, T) \mathrm{d}p_z$$

Energy density $$\rho_e = \frac{eB}{(2\pi)^2} \sum_{n=0}^{\infty} (2 - \delta_{n0}) \int E_B f_{ED}(E_B, T) \mathrm{d}p_z$$

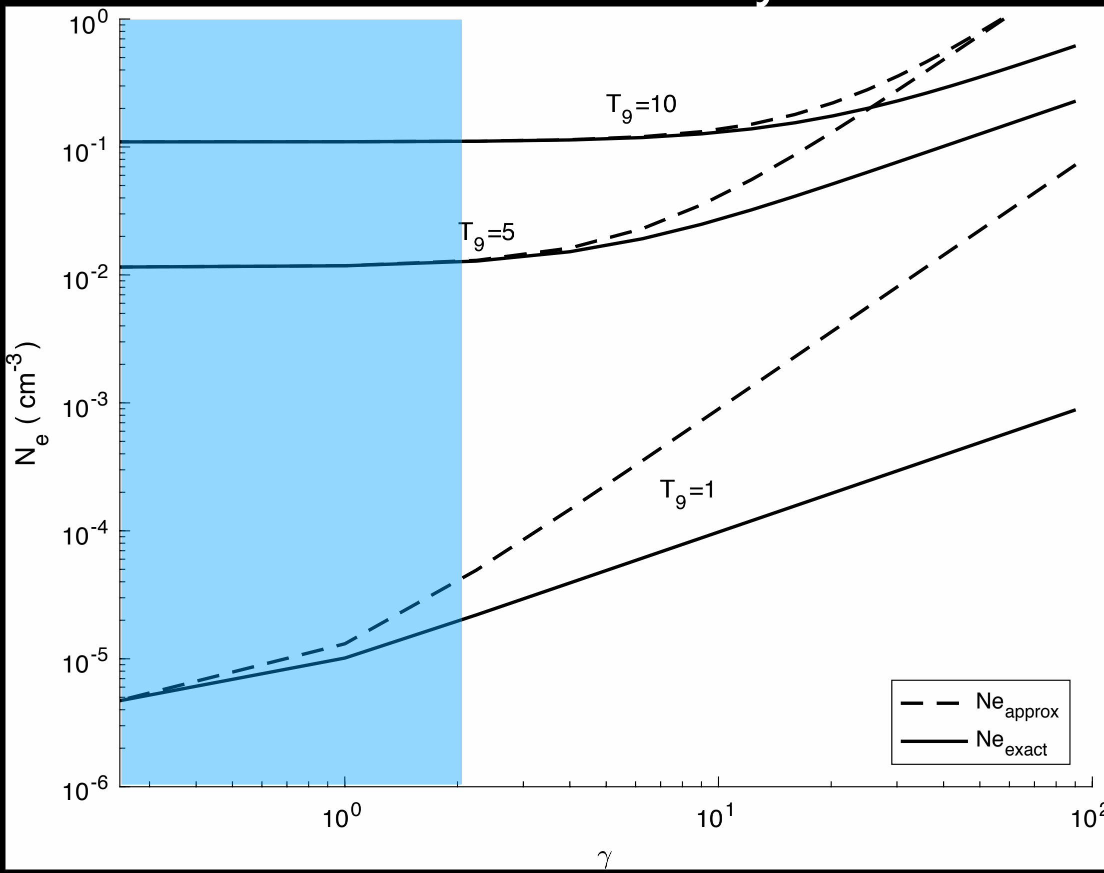
Pressure density $$P_e = \frac{eB}{(2\pi)^2} \sum_{n=0}^{\infty} (2 - \delta_{n0}) \int \frac{E_B^2 - m_e^2}{3E_B} f_{ED}(E_B, T) \mathrm{d}p_z$$

Magnetic field and its impact on thermodynamics

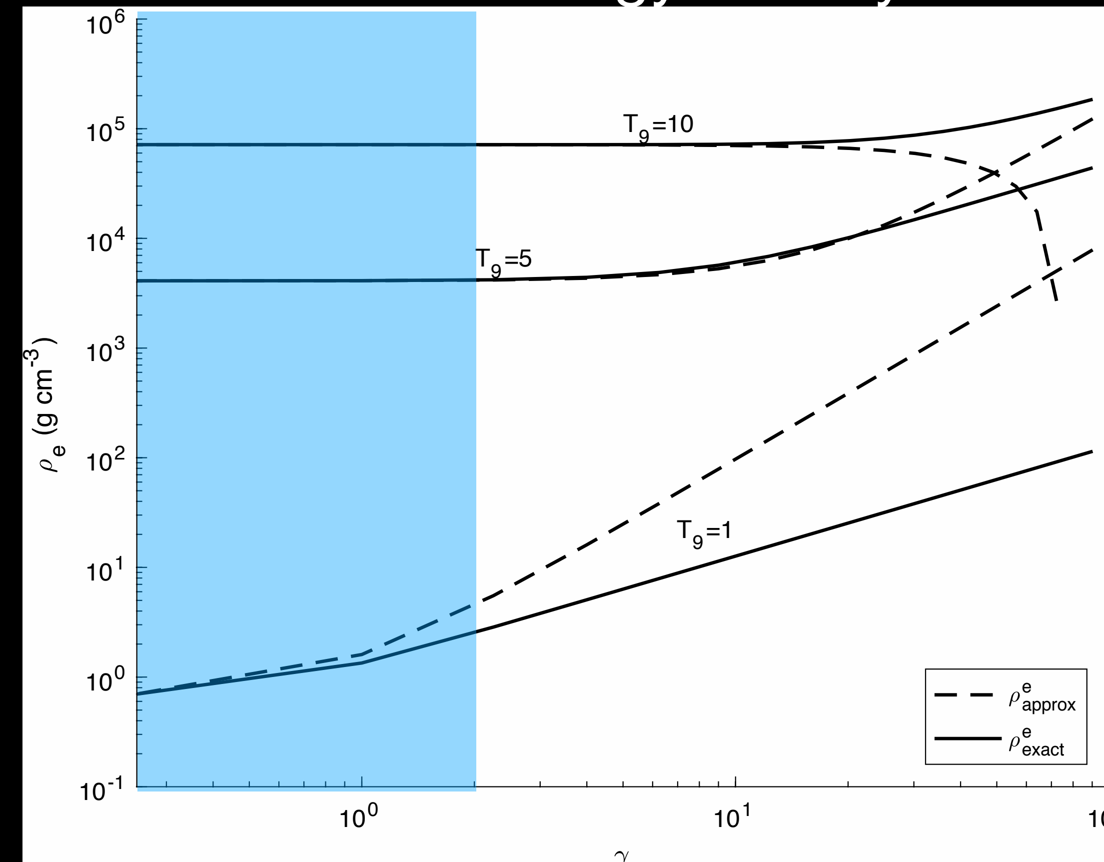
$\gamma := B/B_c$

$B_c = 4.41 \times 10^{13} G$

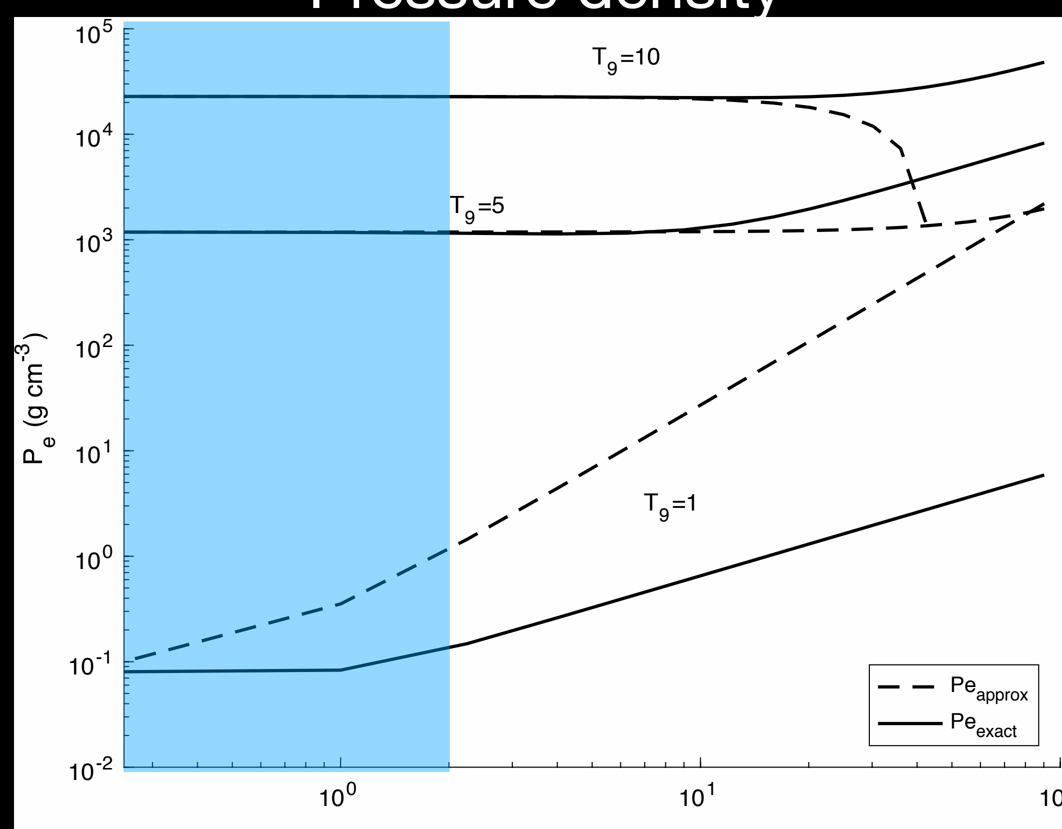
Number density



Energy density



Pressure density



Magnetic field and its impact on time-temperature relation

Friedmann Equations

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 = \frac{8\pi G}{3}\rho_{\text{tot}}$$

$$\frac{d\rho}{dt} = -3H\left(\rho + p\right)$$

H:Hubble expansion rate
a: Scale factor

$\longrightarrow$

- Neutrino temperature

$$\frac{dT_\nu}{dt} = -HT_\nu$$

- Photon temperature

$$\frac{dT_\gamma}{dt} = -3H\frac{\rho_e + \rho_\gamma}{d\rho_\gamma/dT_\gamma + d\rho_e/dT_\gamma}$$

B-field changes thermodynamics of $e^{\pm}$

- $\rho_B \propto \gamma^2 \longrightarrow \rho_{total} = \rho_e + \rho_b + \rho_g + \rho_B \longrightarrow H\uparrow$

- $\rho_e = \rho_e(T_\gamma, \gamma) \longrightarrow \dfrac{d\rho_e}{dT_\gamma} = \dfrac{\partial\rho_e}{\partial T_\gamma} + \dfrac{\partial\rho_e}{\partial\gamma}\dfrac{d\gamma}{dT_\gamma}$

$$\longrightarrow \quad \frac{dT_\gamma}{dt} = -3H\frac{\rho_e + \rho_\gamma}{d\rho_\gamma/dT_\gamma + h}\left(1 - \frac{j}{3(\rho_e + \rho_\gamma)}\right)$$

h : a complicated equation $\quad$ j : another complicated equation

Magnetic field and its impact on weak interaction

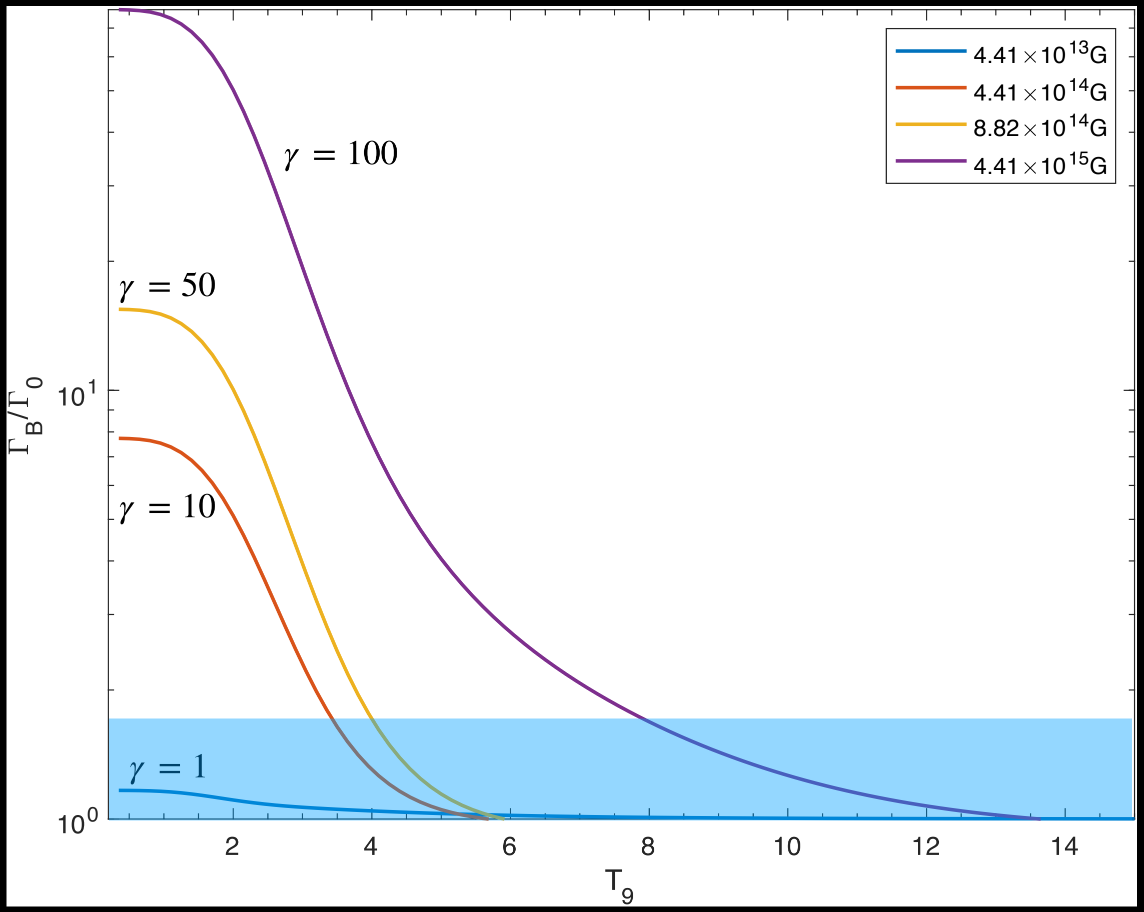
Weak Interaction

$$n + \nu \longleftrightarrow p + e^-$$

$$n \longleftrightarrow p + \bar{\nu} + e^-$$

$$n + e^+ \longleftrightarrow p + \bar{\nu}$$

Comparison of reaction rate w/ and w/o B-field



Interaction rate Γ

$$\Gamma_{n\to p}(B) = \frac{g_V^2(1+3\alpha^2)m_e^5c^4\gamma}{2\pi^3\hbar^7}\sum_{n=0}^{\infty}\left[2-\delta_{n0}(1-P\Lambda)\right]\times\int_{\sqrt{1+4\gamma n}}^{\infty}\epsilon d\epsilon\frac{[\epsilon^2-(1+4\gamma n)]^{-1/2}}{1+\exp(\epsilon Z_\nu+\phi_e)}\times$$

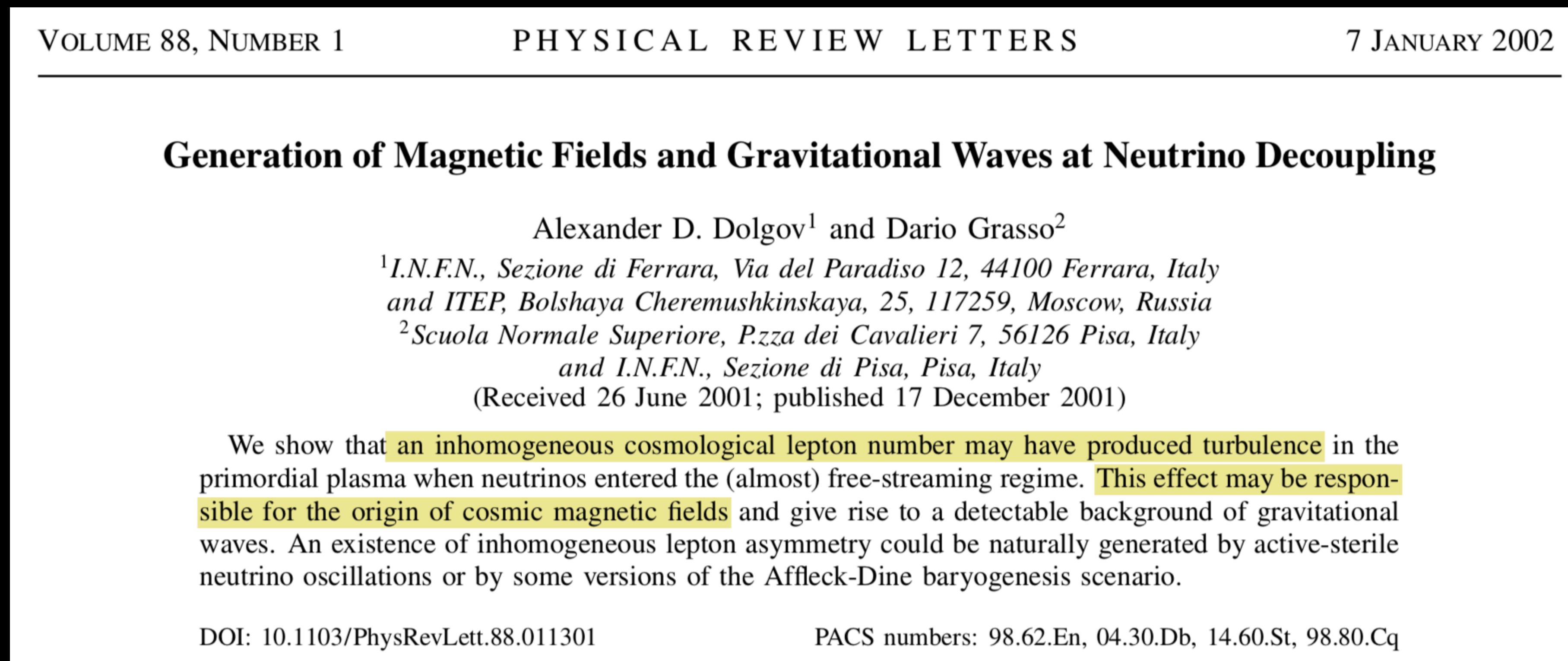
$$\left(\frac{(\epsilon+q)^2\exp[(\epsilon+q)Z_\nu+\phi_\nu]}{1+\exp[(q+\epsilon)Z_\nu+\phi_\nu]}-\frac{(\epsilon-q)^2\exp(\epsilon Z_e+\phi_e)}{1+\exp[(\epsilon-q)Z_\nu+\phi_\nu]}\right)$$

$\Gamma_{p\to n}(B)$ has the similar formalism

(Cheng et al 1993)

Inhomogeneous PMF

Inhomogeneous PMF during BBN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Generation of Magnetic Fields and Gravitational Waves at Neutrino Decoupling

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We show that an inhomogeneous cosmological lepton number may have produced turbulence in the primordial plasma when neutrinos entered the (almost) free-streaming regime. This effect may be responsible for the origin of cosmic magnetic fields and give rise to a detectable background of gravitational waves. An existence of inhomogeneous lepton asymmetry could be naturally generated by active-sterile neutrino oscillations or by some versions of the Affleck-Dine baryogenesis scenario.

DOI: 10.1103/PhysRevLett.88.011301 PACS numbers: 98.62.En, 04.30.Db, 14.60.St, 98.80.Cq

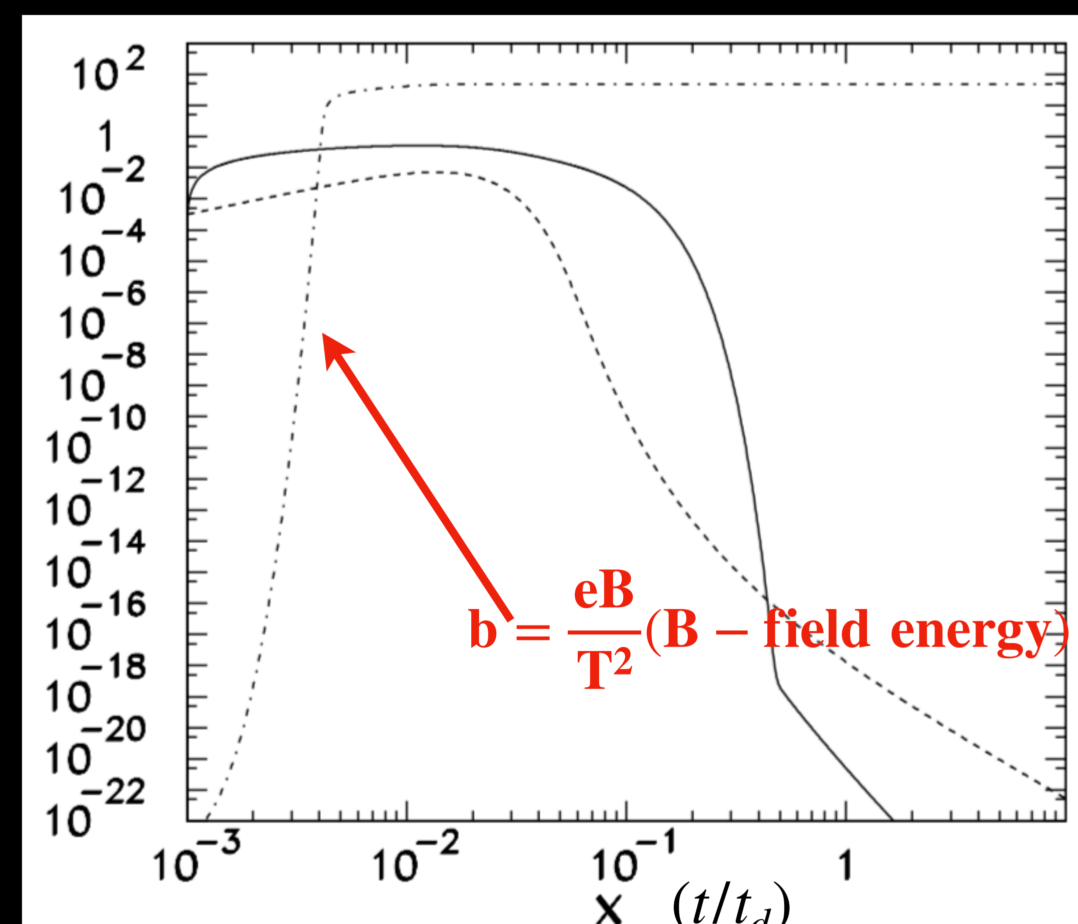
By solving Boltzmann equation of neutrino momentum & Maxwell equation & Euler equation of the fluid:

$$\frac{\partial}{\partial t}K_i(x,t) + 4HK_i(x,t) + \frac{\partial}{\partial x^j}K_{ij}(x,t) = -\tau_w^{-1}K_i$$

$$\partial_t B + 2HB = \nabla \times (\mathbf{v} \times \mathbf{B}) + \kappa^{-1}\nabla \times \mathbf{J}_{ext}$$

$$\frac{\partial \mathbf{v}}{\partial t} \sim \tau_{\nu e}^{-1}\frac{\rho_\nu}{(\rho+p)\gamma^2}(K_\nu + K_{\bar{\nu}}) - H\mathbf{v} + \eta\nabla^2\mathbf{v}$$

$\lambda/H^{-1} = 10^{-2}$

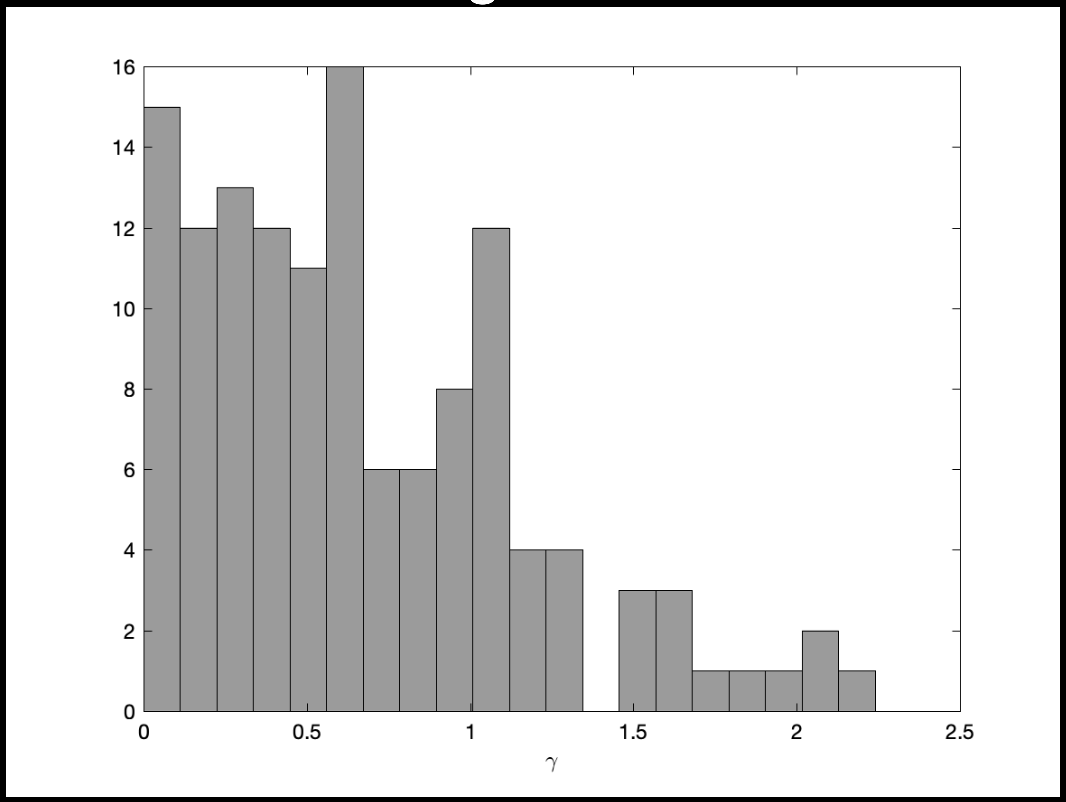


(Dolgov & Grasso 2002)

Multi-Zone BBN (non-dynamical)

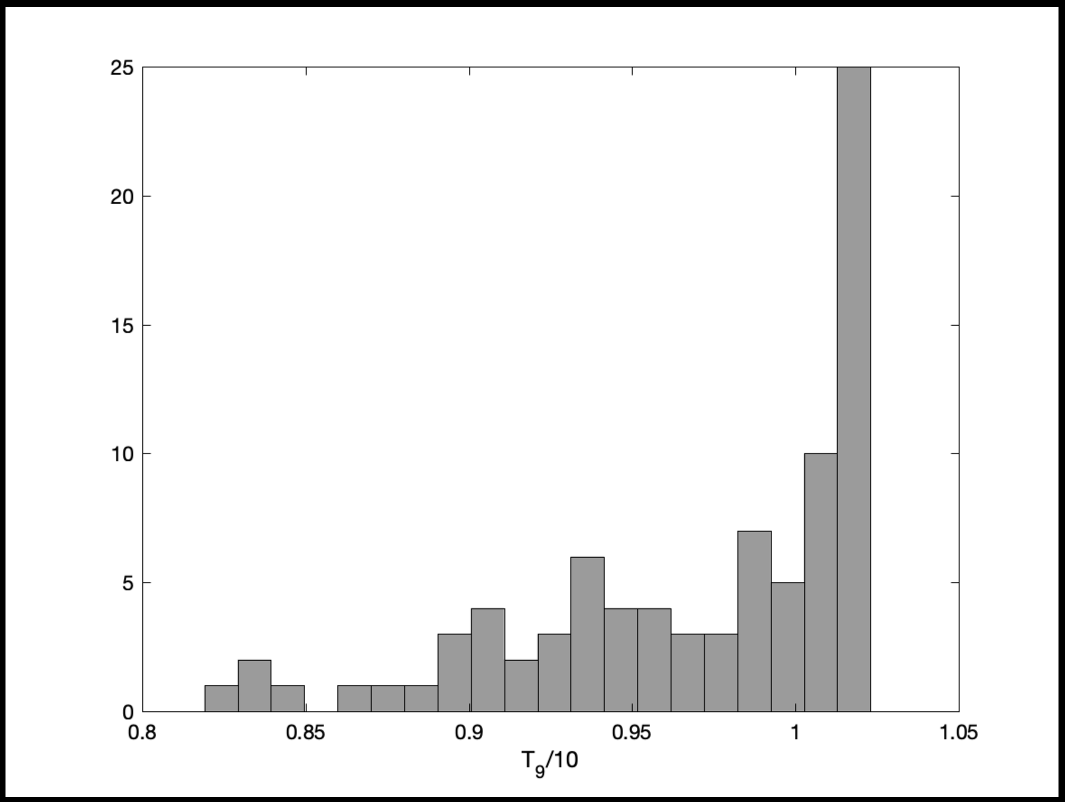
Initial distribution function set-up
@ T9=10

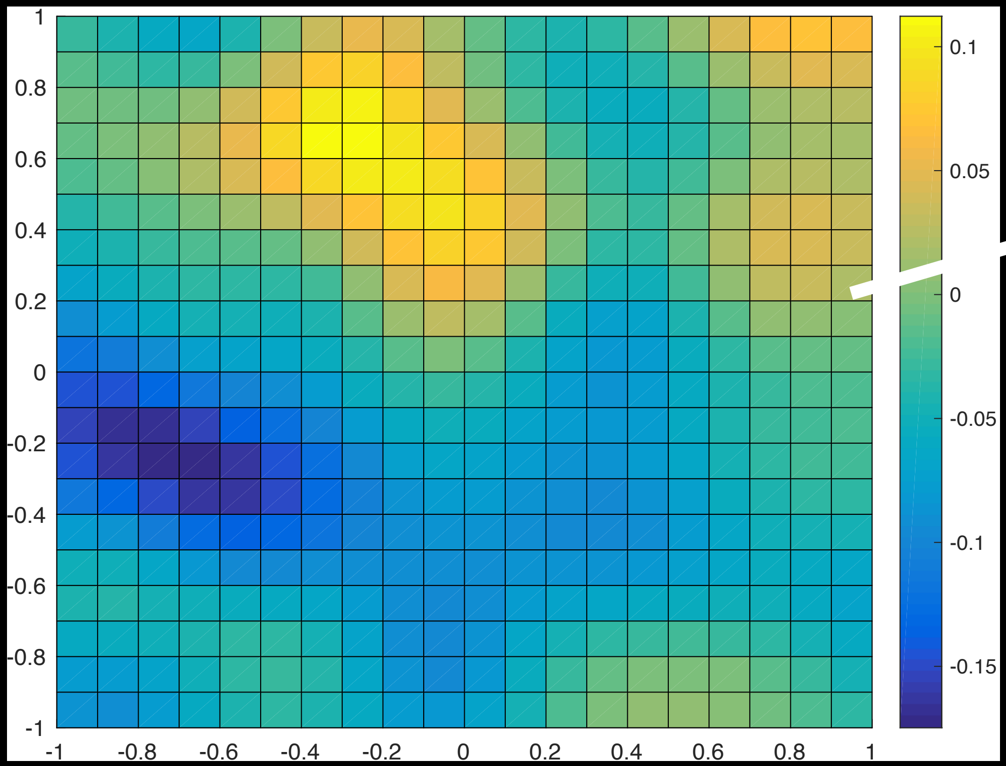
Histogram of γ



$1/T \propto \rho_{rad}^{-1/4}$

Histogram of T_9





Evolve BBN code with

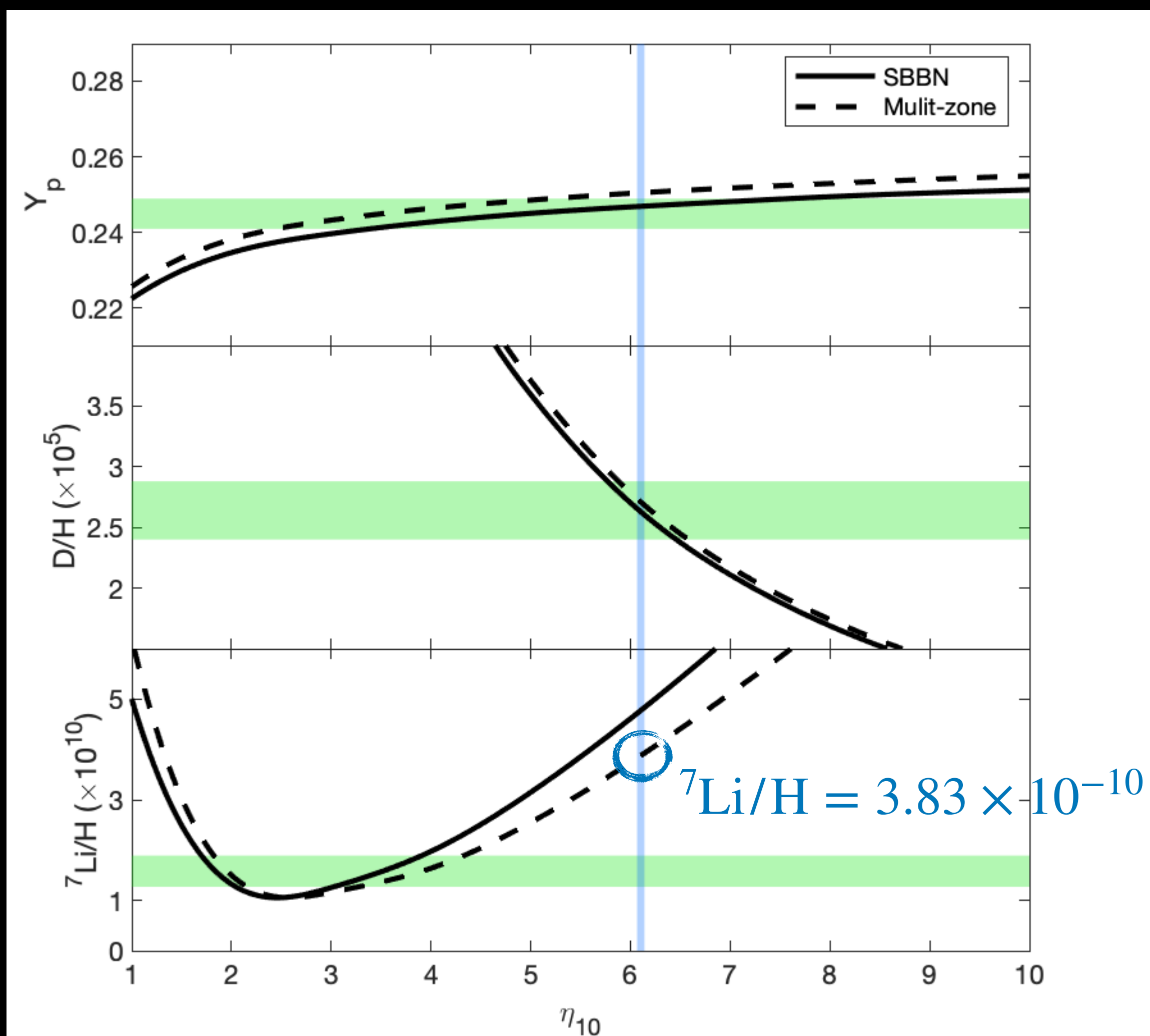
Approximation formula
$n_e(B); \rho_e(B); P_e(B)$

+ Time-temperature relation w/B-filed

+ Weak Interaction w/B-field

Result (non-dynamical)

Multi-zone result
(after averaging all zones)



Previous Study

Y.Luo.; et al ApJ 872,172 (2019)

$$\langle \sigma v \rangle (T') = \int \sigma(E) v f_{MB}(v \mid T') dv$$

Since temperature also has distribution…

$$\langle \sigma v \rangle (T) = \int \langle \sigma v \rangle (T') f(T') dT'$$

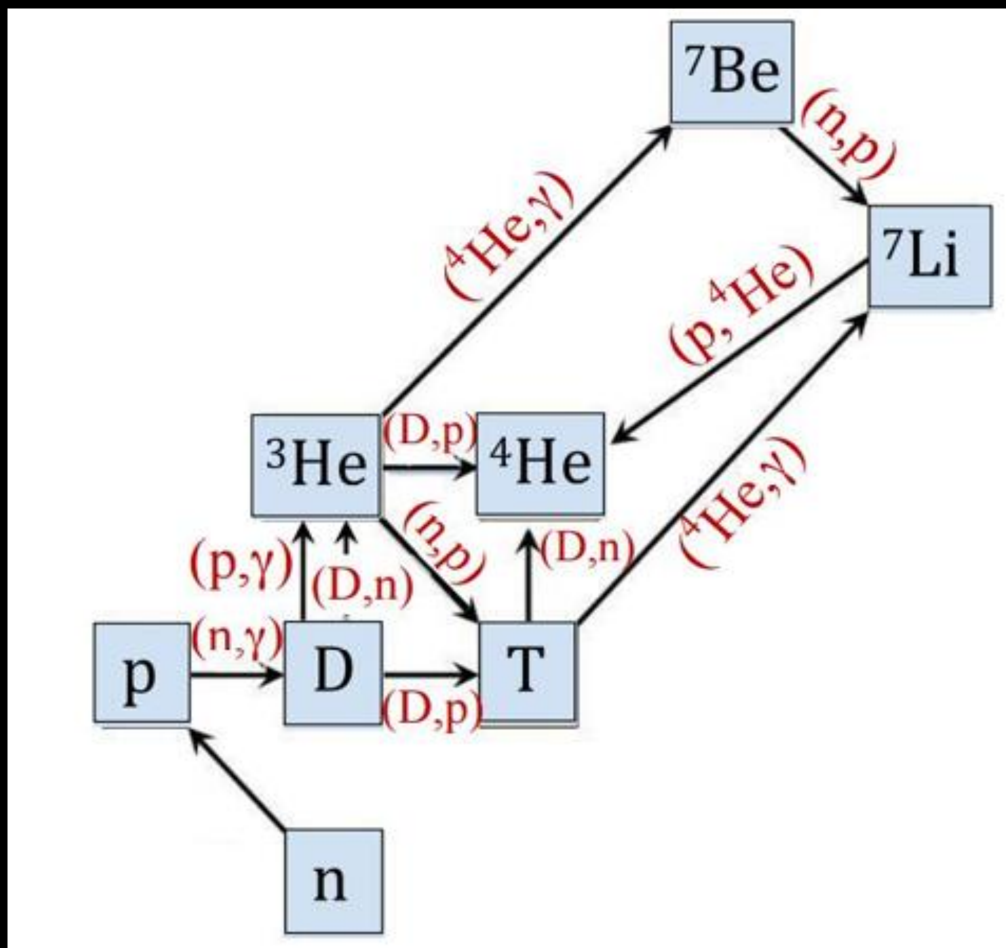
$$= \int \sigma(E) v F(v) dv$$

$$F(v) \equiv \int dT' f(T') f_{MB}(v \mid T')$$

(Effective distribution function)

Next step

BBN reaction network



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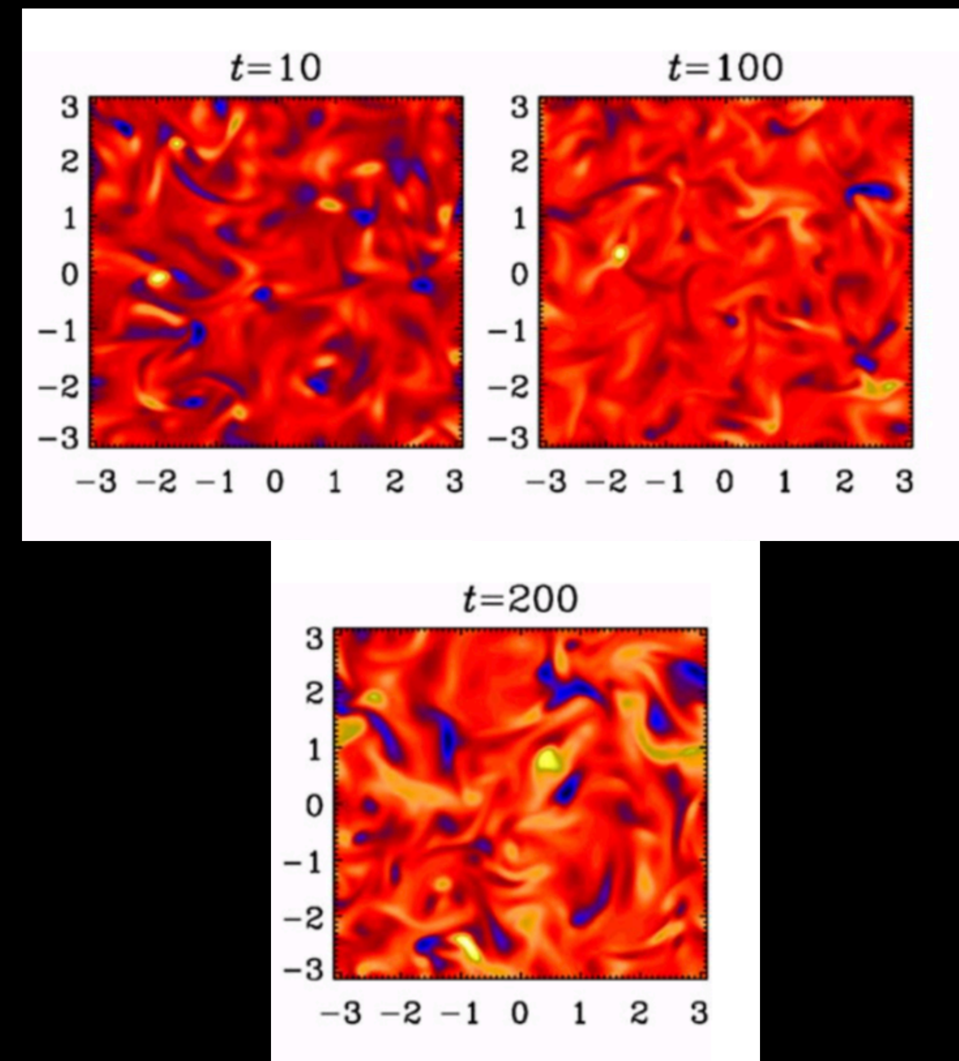
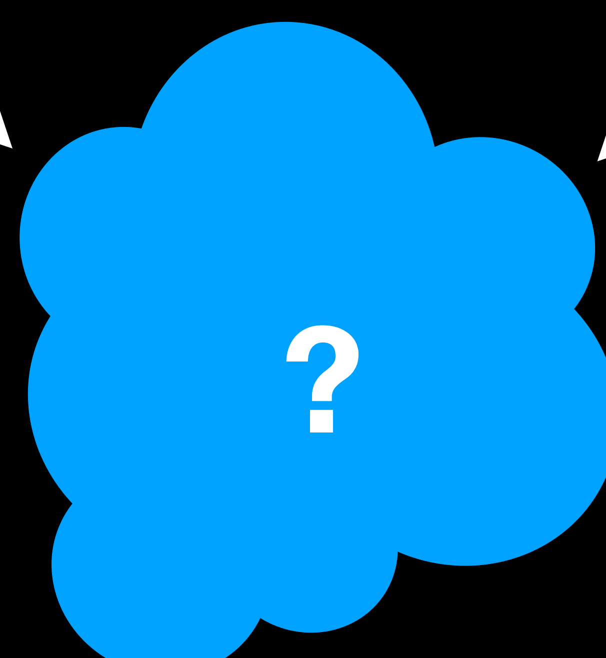
Magnetohydrodynamics (MHD)



(Brandenburg et al 2005)



Validity

- **Density of the plasma:** ⟶ Low density, favor of dynamos

The present day photon number density $\sim 416\ cm^{-3}$ $\eta = n_b/n_\gamma = 6.10 \times 10^{-10}$ $T_0 = 2.73$ K

@T9=1, since $a \sim 1/T$ ⟶ $n_b \sim 10^{19}$ cm^{-3}

take account only proton, $m_p = 1.67 \times 10^{-24}$ g ⟶ $\rho_{Baryon}(T_{BBN}) \sim 10^{-5}$g cm^{-3}

- **Reynolds number** ⟶ Perfect conductor; Also turbulent is possible

$R_e \sim 10^{11}$; $R_{e_M} \sim v\sigma H^{-1} \sim M_P/T \sim 10^{17}$ ⟶ Prandtl number is large

(Son et al 1999)

- **MHD Formalism in an expansion Universe**

$$\frac{\partial \mathbf{v}}{\partial \tau} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla\left(\tilde{p} + \frac{\tilde{B}^2}{2}\right) + (\tilde{\mathbf{B}} \cdot \nabla)\tilde{\mathbf{B}} + \tilde{\nu}\Delta\mathbf{v}$$

(Gailis et al 1995, Brandenburg et al 1996,Son 1999,Christensson et al 2001)

$$\frac{\partial \tilde{\mathbf{B}}}{\partial \tau} = \nabla \times (\mathbf{v} \times \tilde{B}) + \tilde{\eta}\Delta\tilde{\mathbf{B}}$$

$\tau = \int dta^{-1}(t)$ (conforaml time) $\tilde{\mathrm{X}}$: quantity in comoving frame

⟶ Same formalism as MHD in non-expand relativistic fluid except a co-moving frame of reference

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Can Inhomogeneous Primordial Magnetic Field Affect Nucleosynthesis in the Early Universe?

- Magnetic field mainly affects the electron-positron thermodynamics (n_e, ρ_e, P_e), the time-temperature relation and the weak interaction rate during BBN.
- Nuclear reaction rates can be affected due to the temperature inhomogeneity which is induced by the inhomogeneous PMF.
- A multi-zone BBN code has been developed, however, in order to study the dynamics, we need MHD calculation during BBN.

Thank you

11.Sep.2019@St. Petersburg

Our assumption

(ApJ 872,172)

——Temperature inhomogeneous induced by magnetic field strength fluctuation

$$f(\rho_B) = \frac{1}{\sqrt{2\pi}\sigma_B}\exp\left[-\frac{(\rho_B - \rho_{Bc})^2}{2\sigma_B^2}\right]$$

$\rho_{tot} = \rho_B + \rho_{rad}$ *is homogeneous*

$$T \propto \rho_{rad}^{-1/4} \longrightarrow 1/T \propto \left[\rho_{tot} - \rho_B\right]^{-1/4}$$

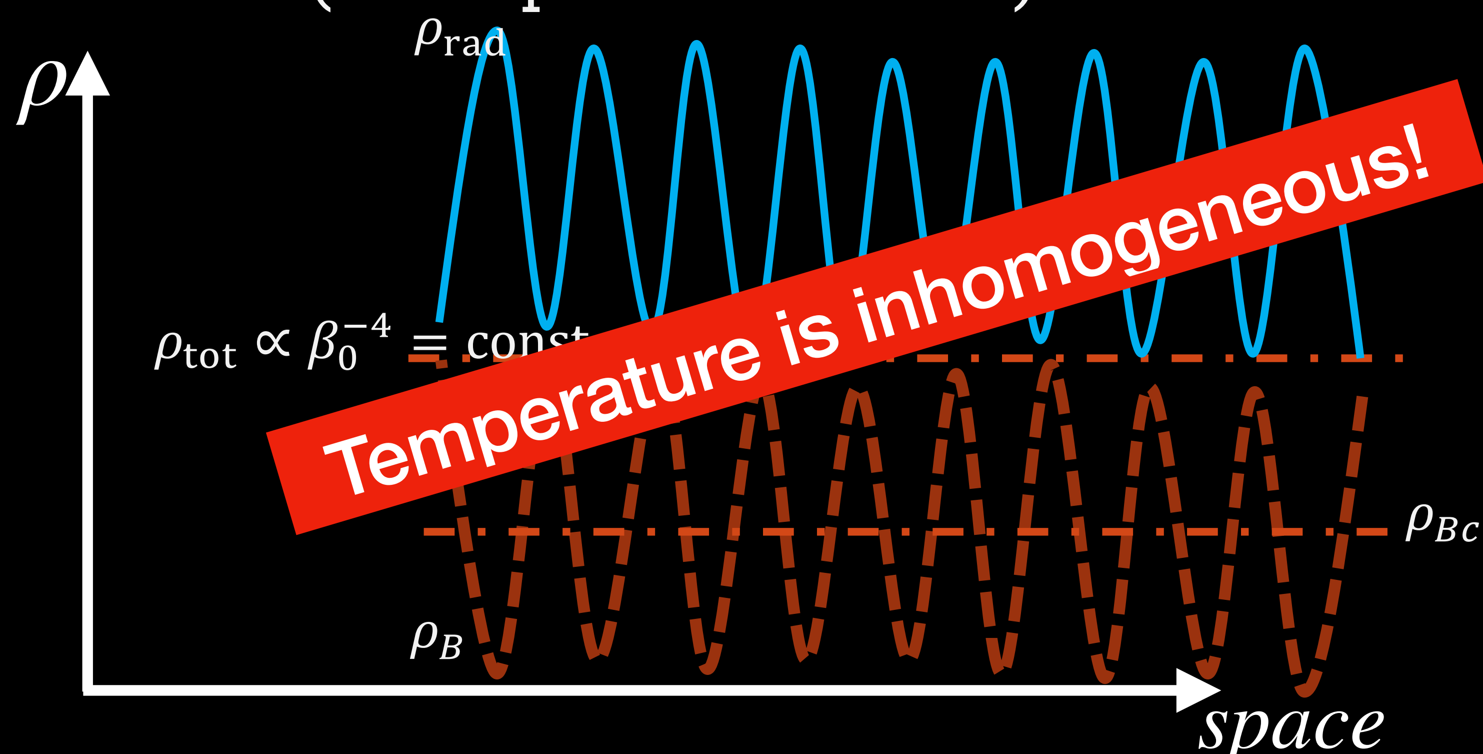
ρ_{Bc} : $\frac{\langle B^2 \rangle}{8\pi}$ Mean value from previous study (Yamazaki et al 2012)

σ_B : Decide the width

Temperature distribution

$$f(T) = \frac{1}{\sqrt{2\pi}\sigma_B}\exp\left[-\frac{(\frac{\pi g_*}{30}(T_{tot}^4 - T^4) - \rho_{Bc})^2}{2\sigma_B^2}\right]\frac{2\pi g_*}{15}T^3$$

(Out of phase fluctuation)



(ApJ 872,172)

Main Effect:
——Non-Maxwellian distribution of nuclei

Reaction rate: $\langle\sigma v\rangle(T)$

$$\langle\sigma v\rangle(T') = \int \sigma(E) v f_{\rm MB}(v\,|\,T') dv$$

$\sigma(E)$: *cross section* v : *relative velocity*

$f_{\rm MB}$: *Maxwell – Boltzmann distribution*

$$F(v) \equiv \int dT' f(T') f_{\rm MB}(v\,|\,T')$$

Deviation from Maxwellian distribution!

Since temperature also has distribution…

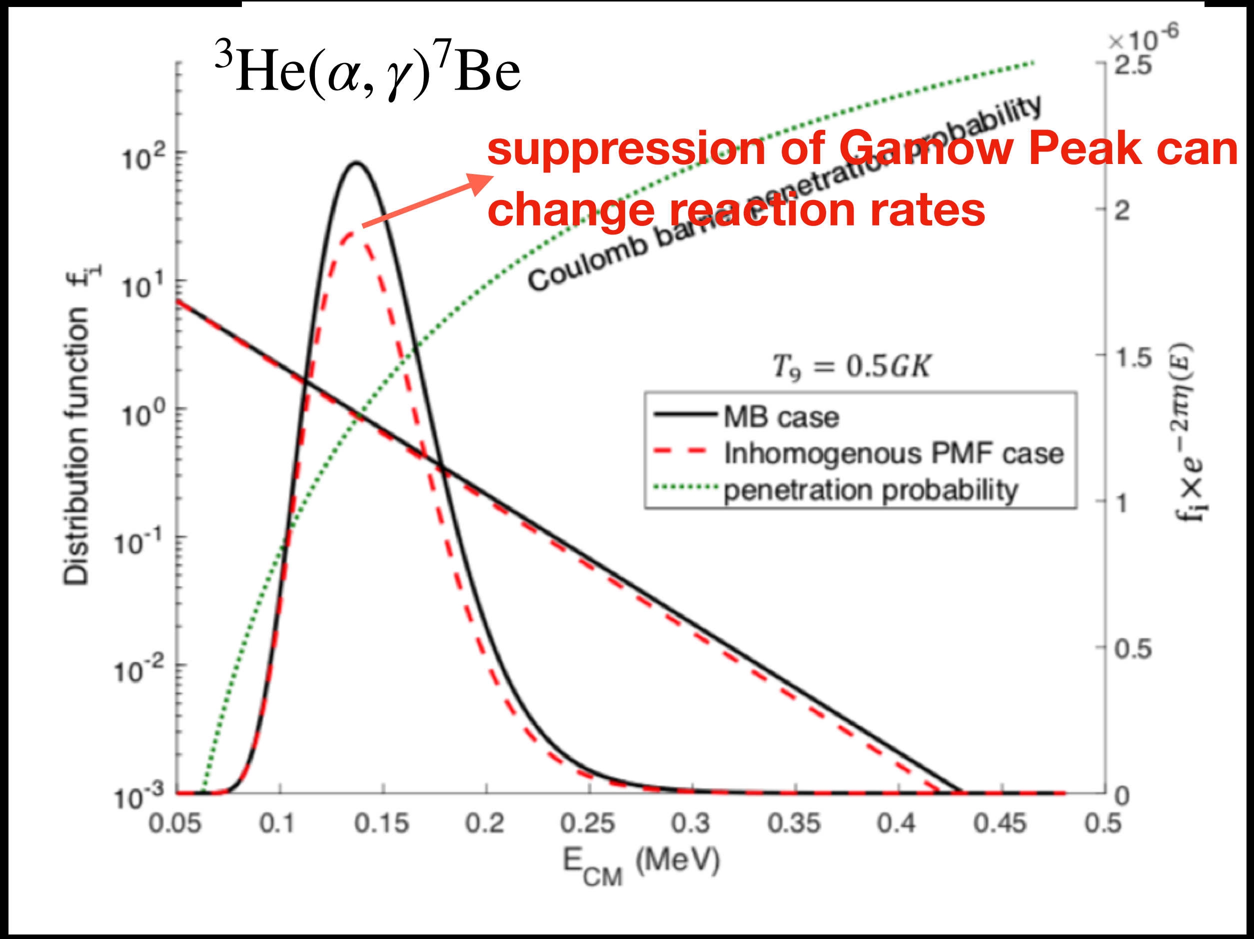
$$\langle\sigma v\rangle(T) = \int \langle\sigma v\rangle(T') f(T') dT' = \int \sigma(E) v F(v) dv$$

Locally nuclei obey a classical MB distribution

Thermonuclear reaction rates averaged over the set of temperature fluctuations

$f(T')$: *distribution function of* T'

For charged nuclei reaction

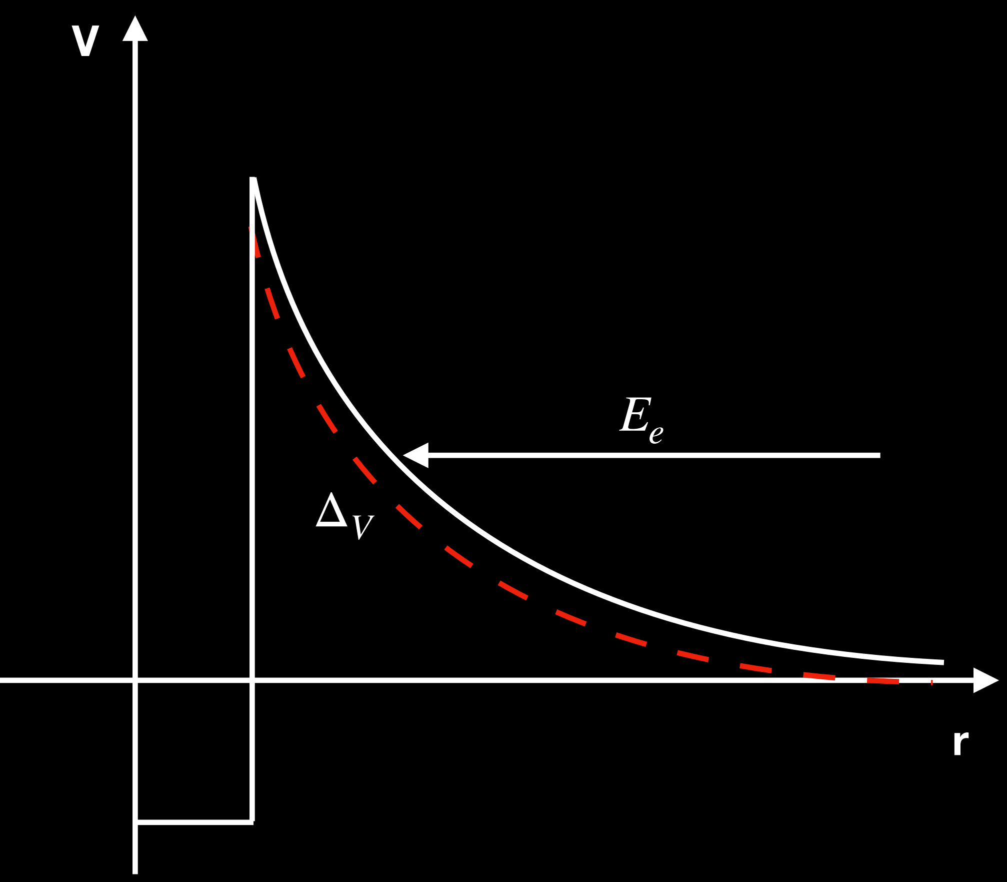


Magnetic field and its impact on screening effect

Electron Capture

$$p + e^- \longleftrightarrow n + \nu \quad n + e^+ \longleftrightarrow p + \bar{\nu}$$

$$\lambda_{pe^- \to n\nu_e} = \frac{G_F^2 T_\gamma^2 (g_V^2 + 3g_A^2)}{2\pi^3} z \sum_{n=0}^{\infty} (2 - \delta_{n0}) \int_0^\infty dp_z E_\nu^2 g(E_e/T_e) f_{FD}(E_\nu/T_\nu)$$



$$V_{scr} = \frac{Z_1 Z_2 e^2}{r} \exp\left(-\frac{r}{\lambda}\right) \sim V_{col} - \frac{Z_1 Z_2 e^2}{\lambda}$$

$$E'_e = E_e - \Delta_V$$

$$\frac{\pi^2}{\lambda^2} = e^2 \frac{\gamma m_e^2}{2} \frac{\partial}{\partial \mu} \sum_{n=0}^{\infty} (2 - \delta_{n_0}) \times$$

$$\int_0^\infty dp_z \left[\frac{1}{1 + \exp(E_n - \mu)/kT} - \frac{1}{1 + \exp(E_n + \mu)/kT}\right]$$

Weak screening effect, not suitable for degeneracy electron gas (Neutron star)