Symbolic dynamics, enthropy and mixing in the free-fall equal-mass three-body problem

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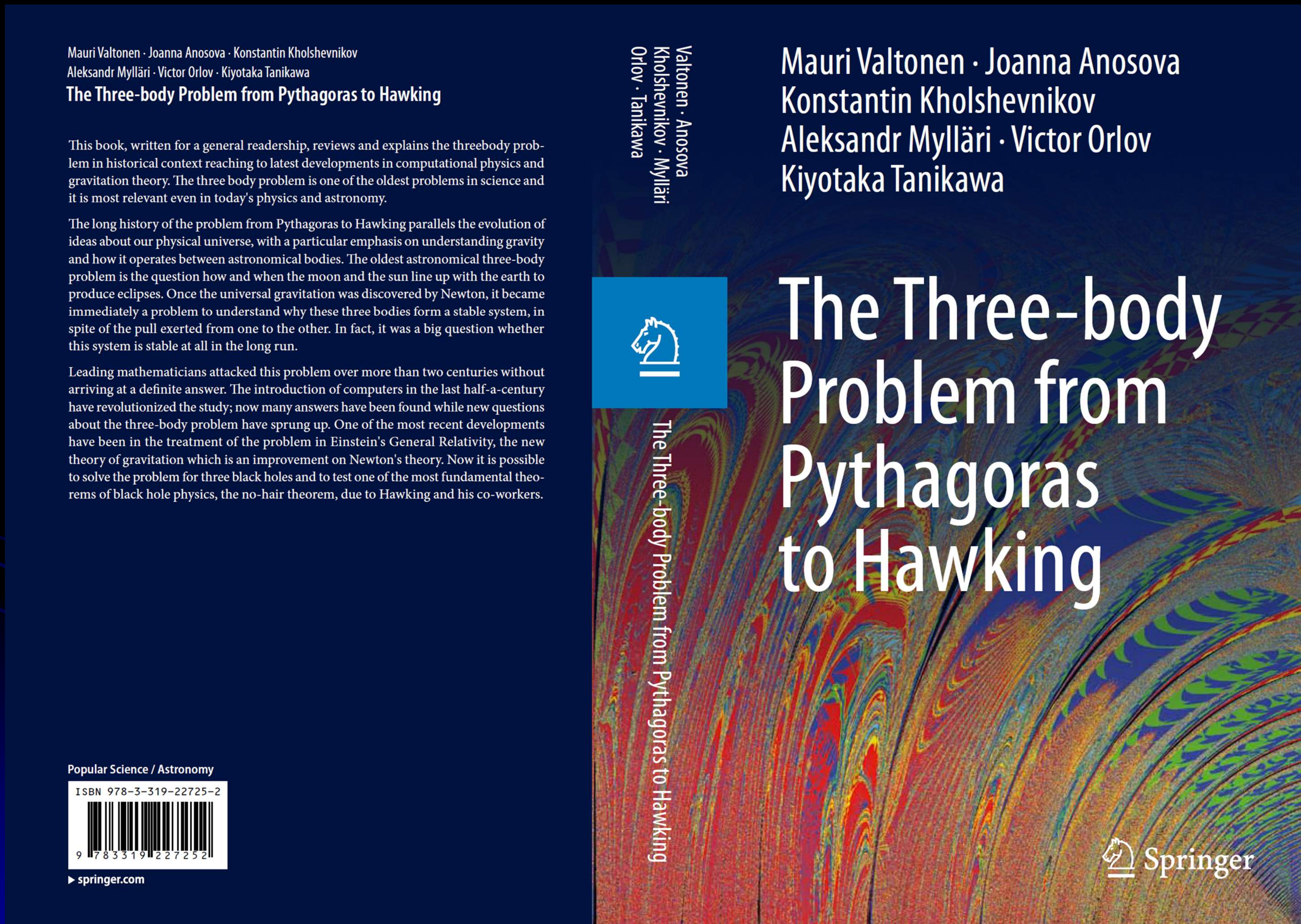
Motivation


Mauri Valtonen · Joanna Anosova · Konstantin Kholshevnikov
Aleksandr Mylläri · Victor Orlov · Kiyotaka Tanikawa
The Three-body Problem from Pythagoras to Hawking
This book, written for a general readership, reviews and explains the threebody problem in historical context reaching to latest developments in computational physics and gravitation theory. The three body problem is one of the oldest problems in science and it is most relevant even in today's physics and astronomy.
The long history of the problem from Pythagoras to Hawking parallels the evolution of ideas about our physical universe, with a particular emphasis on understanding gravity and how it operates between astronomical bodies. The oldest astronomical three-body problem is the question how and when the moon and the sun line up with the earth to produce eclipses. Once the universal gravitation was discovered by Newton, it became immediately a problem to understand why these three bodies form a stable system, in spite of the pull exerted from one to the other. In fact, it was a big question whether this system is stable at all in the long run.
Leading mathematicians attacked this problem over more than two centuries without arriving at a definite answer. The introduction of computers in the last half-a-century have revolutionized the study; now many answers have been found while new questions about the three-body problem have sprung up. One of the most recent developments have been in the treatment of the problem in Einstein's General Relativity, the new theory of gravitation which is an improvement on Newton's theory. Now it is possible to solve the problem for three black holes and to test one of the most fundamental theorems of black hole physics, the no-hair theorem, due to Hawking and his co-workers.
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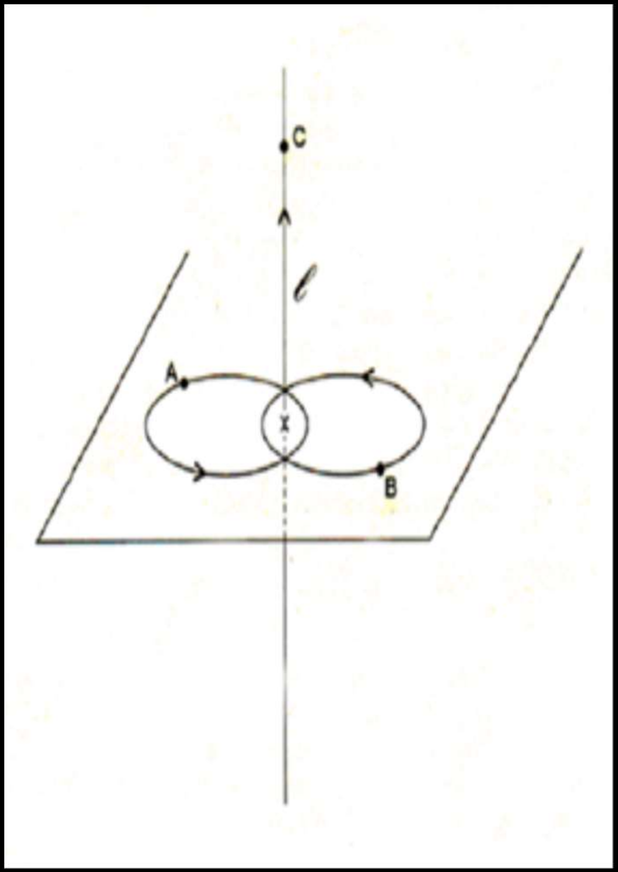
Springer

History

Sitnikov problem
(Alexeev, 1969)
Rectilinear problem
(Tanikawa & Mikkola, 2000)
Isosceles problem
(Zare & Chesley, 1998; Chesley, 1999)
Free-fall equal-mass
three-body problem
(Chernin et al., Mylläri et al., 2004, 2006)

Sitnikov problem

(Alexeev, 1969)



Basic Ideas of Symbolic Dynamics

Symbolic dynamical system **consists of three parts:**

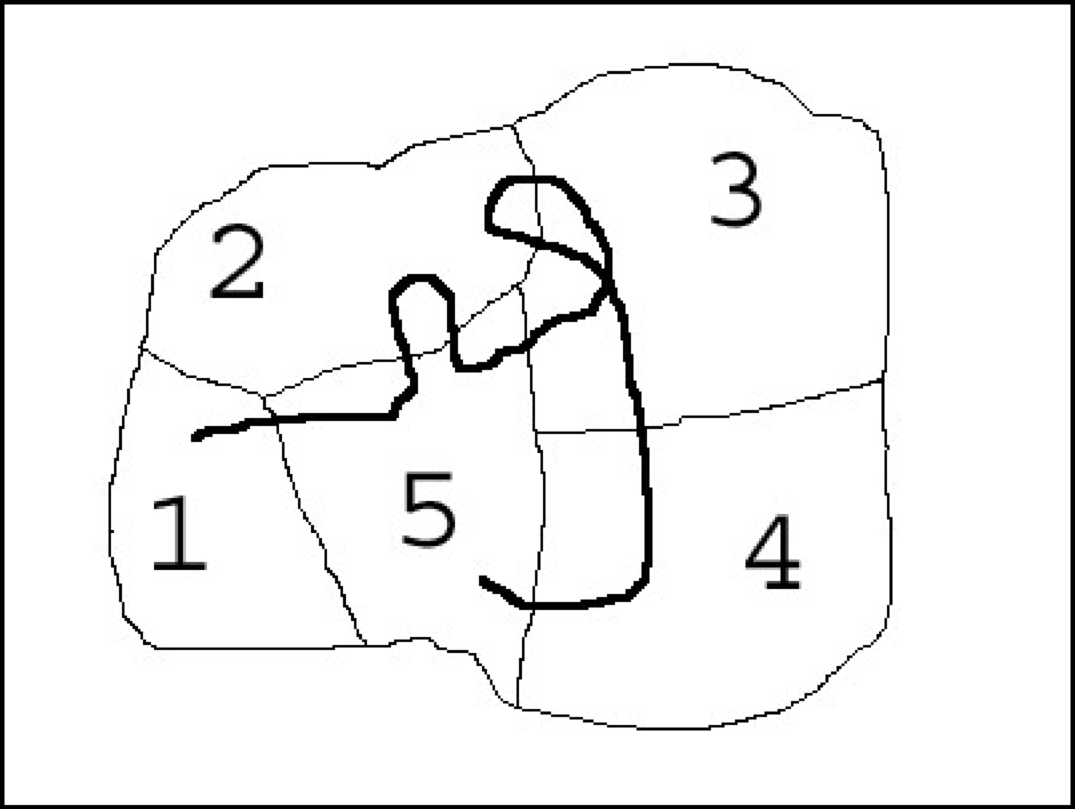
Ω – finite alphabet;

X – space of infinite sequences

$\{\omega_i\}, \; i \in Z, \;\; \omega_i \in \Omega$;

σ – shift transformation,

σ : $\{\omega_i\}'$, ω'_i = ω_{i+1}



…1, 5, 2, 5, 3, 2, 3, 4, 5,…

Examples of Symbolic Sequences for $\Omega=\{0, 1\}$

- Fixed Point

… 0, 0, 0, 0, 0, 0, …

- Trajectory coming to Fixed Point

… 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, …

- Periodic Trajectory

… 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, …

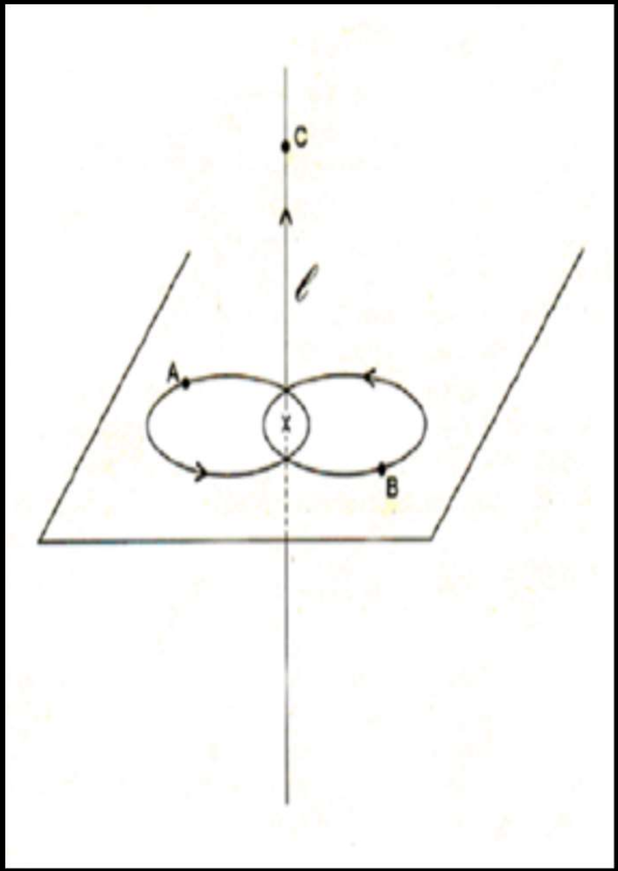
- "Dense" Trajectory

… 0,1,0,0,0,1,1,0,1,1,0,0,0,0,0,1,0,1,0,0,1,1,1,0,1,1,1,0,1,1,1,…

(all combinations of length 1, 2, 3, …)

Sitnikov problem

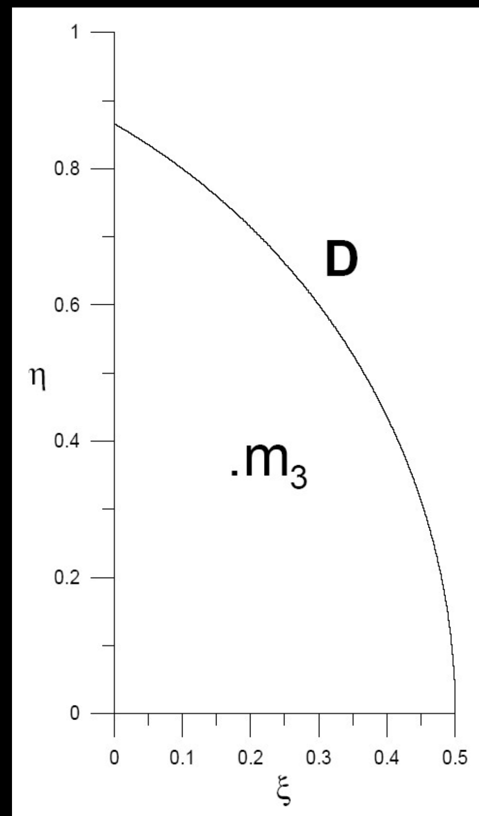
(Alexeev, 1969)



Different ways to construct the symbolic sequence

- **Partitioning of the phase space**
- **Fixing special dynamical states during the evolution of the triple system (double encounters, triple encounters, special configurations, etc.)**

One-to-One Correspondence between Dynamical System and Symbolic Sequence



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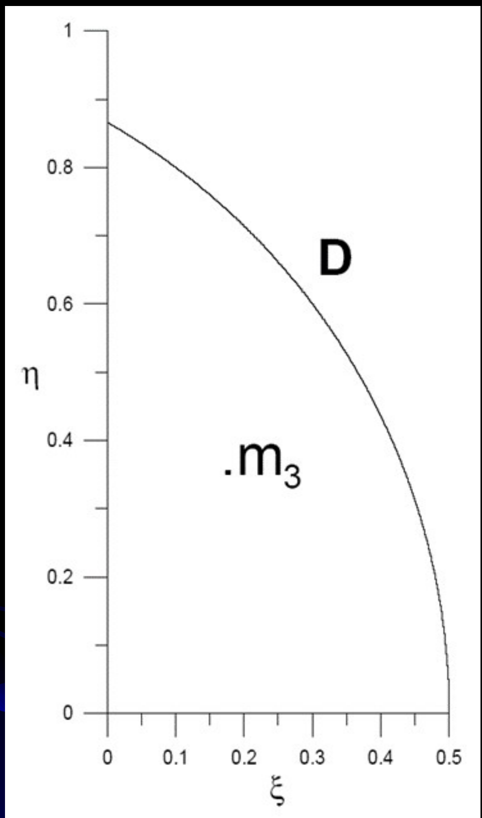

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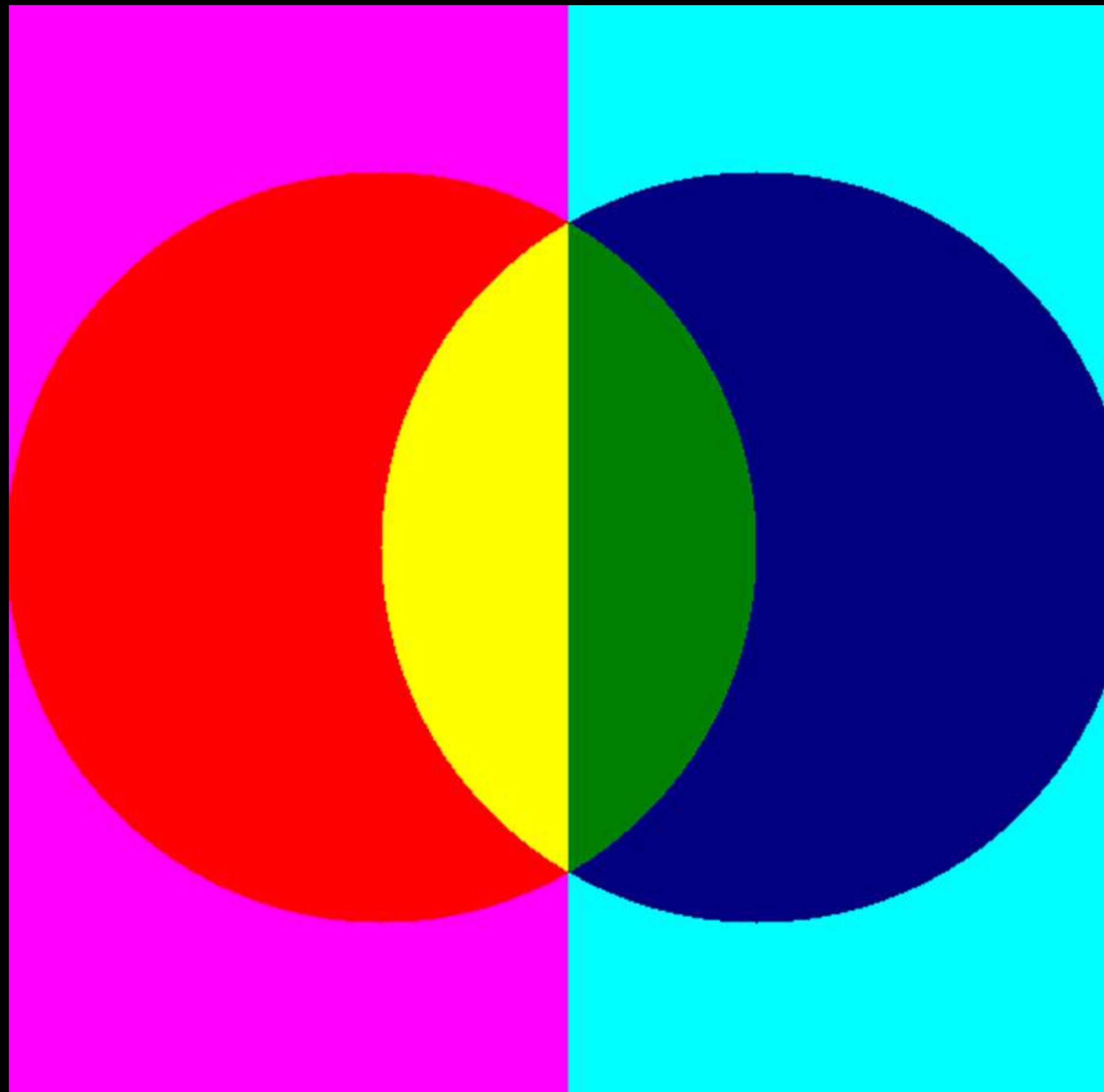
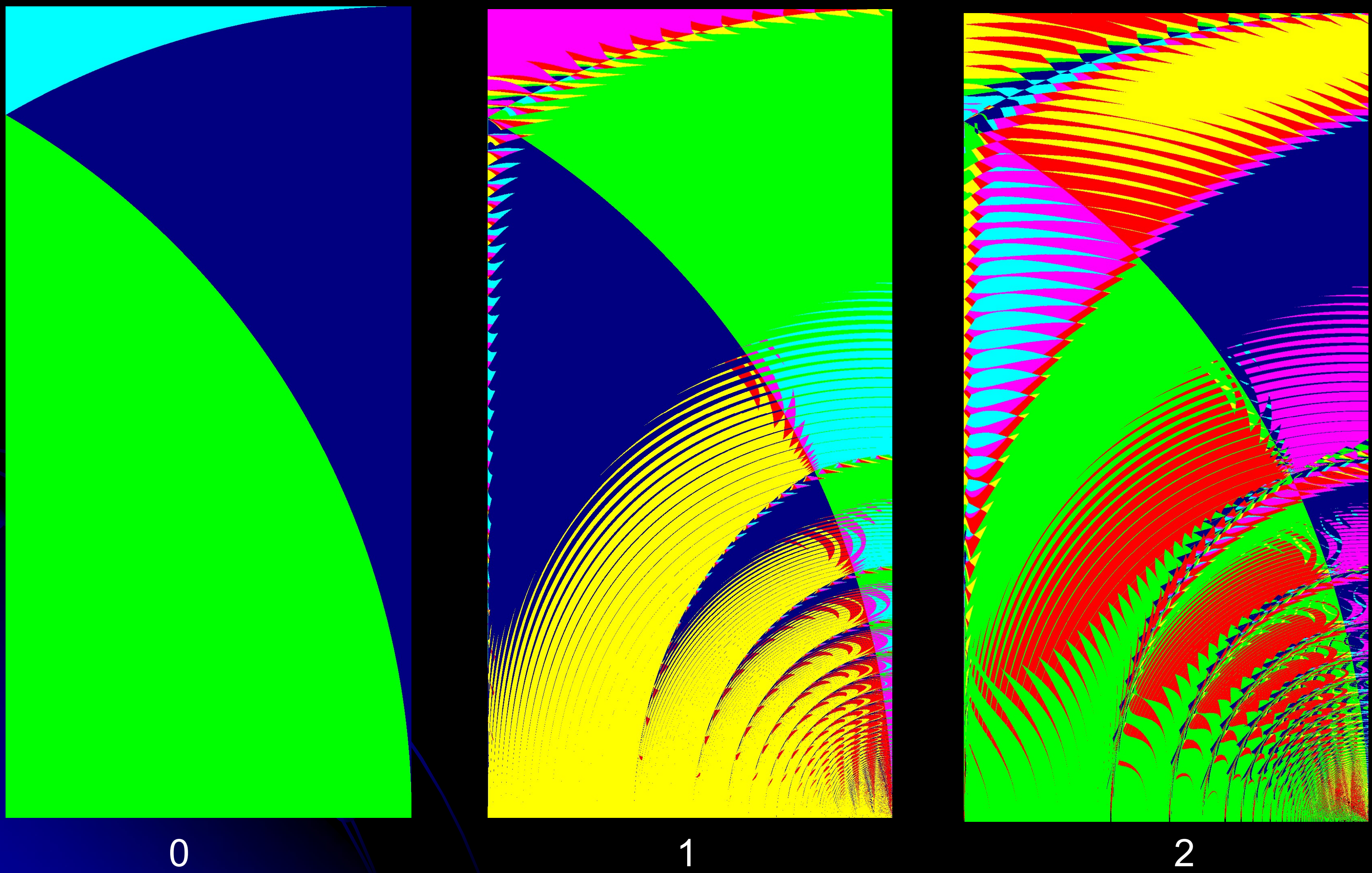
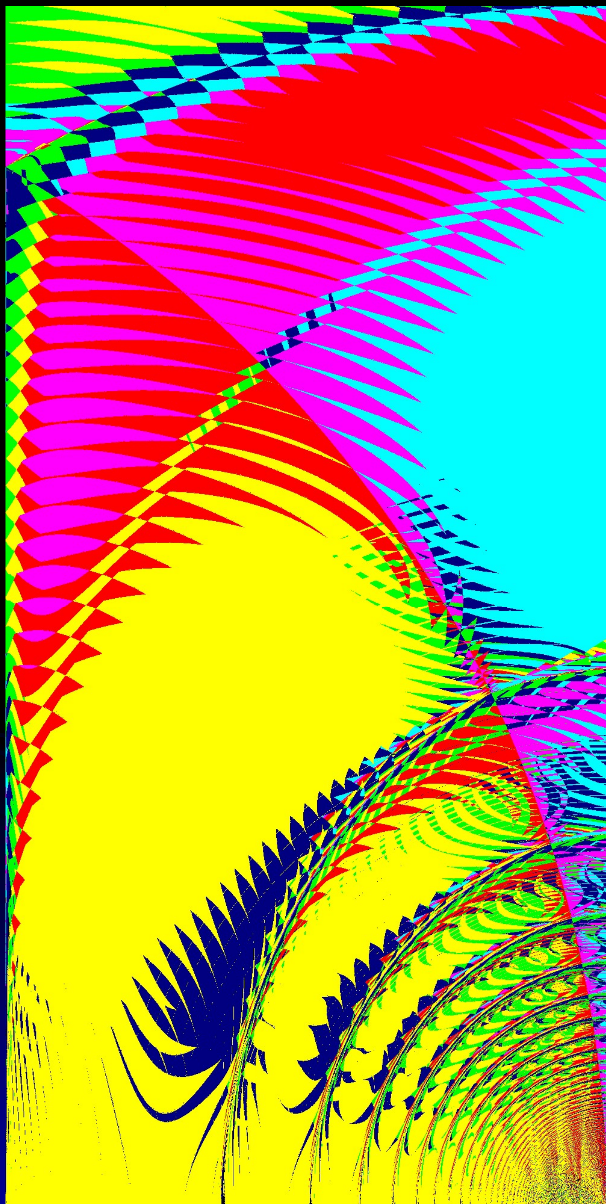


One-to-One Correspondence between Dynamical System and Symbolic Sequence

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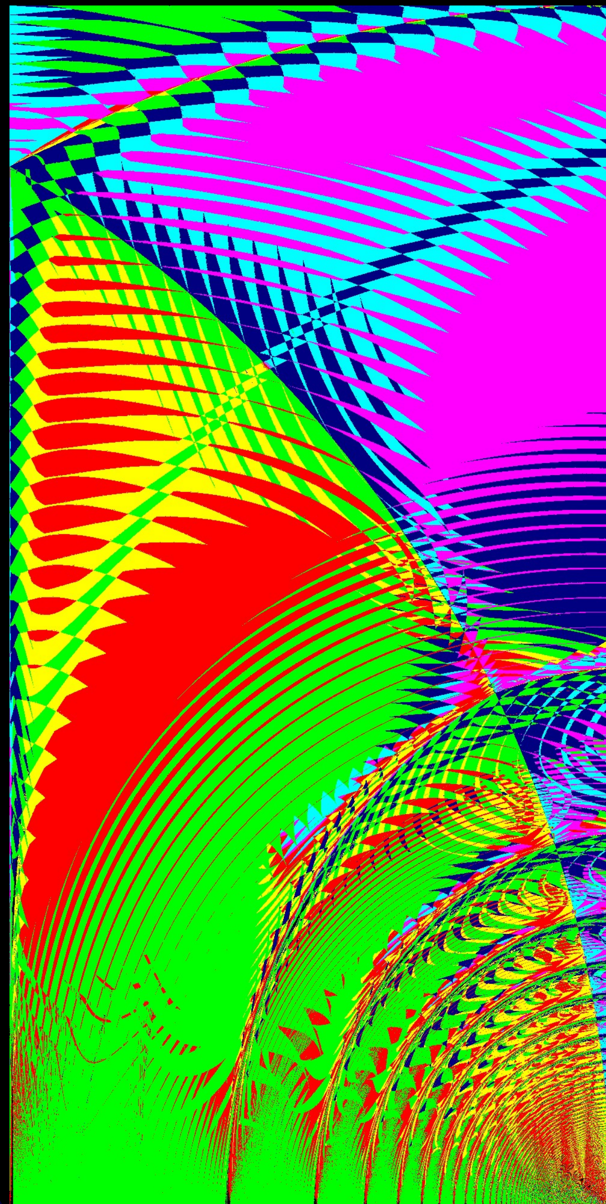
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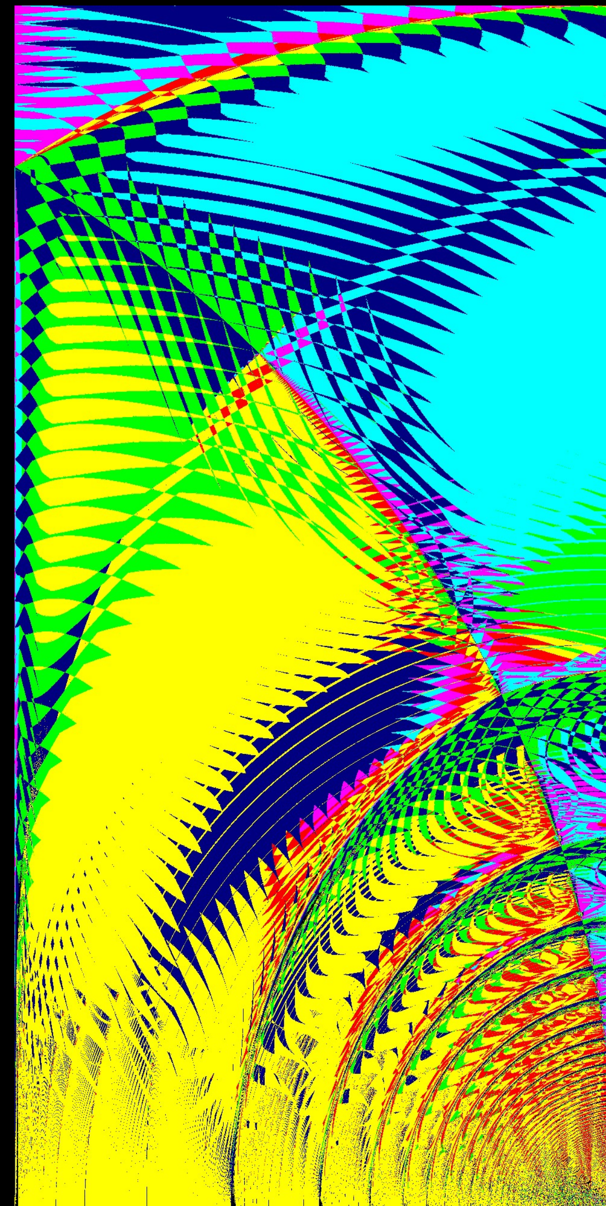
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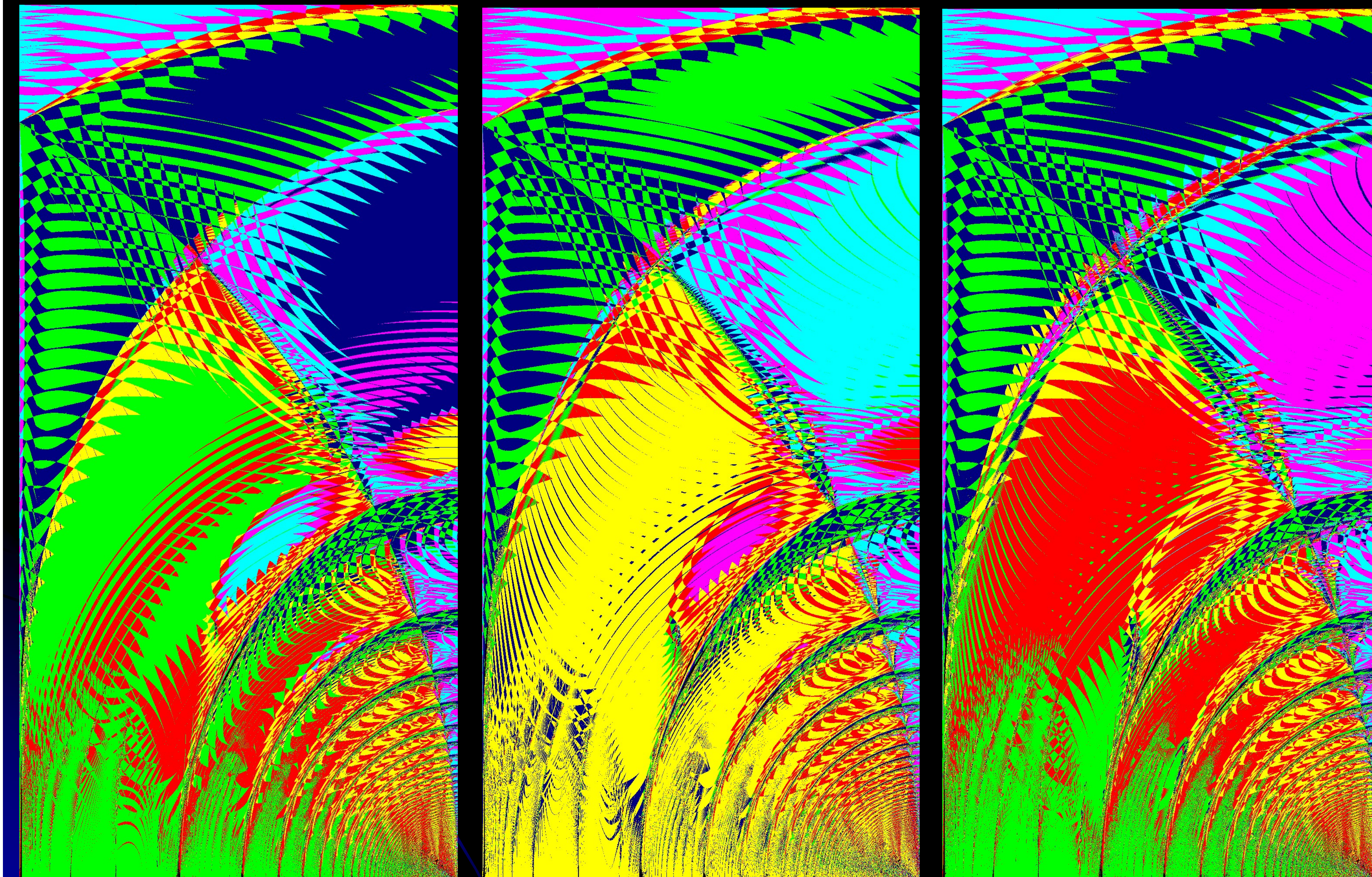

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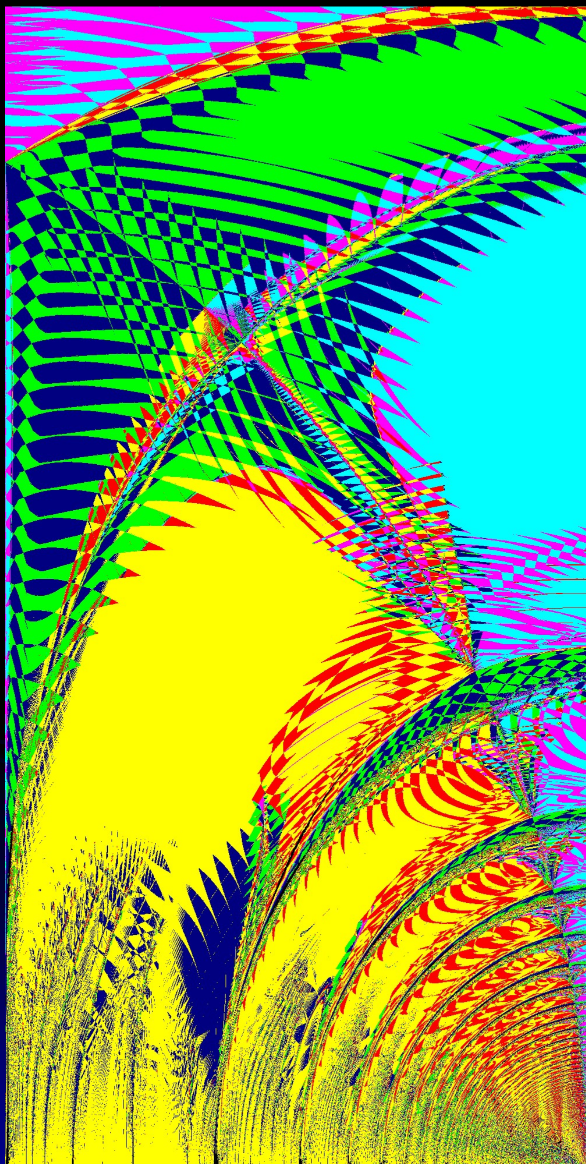

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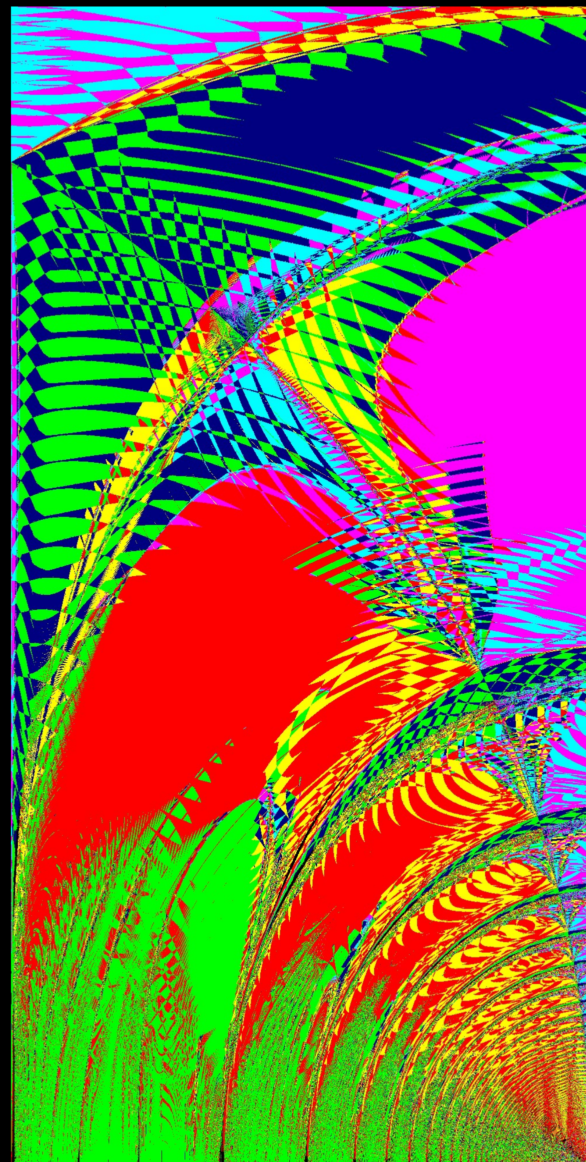

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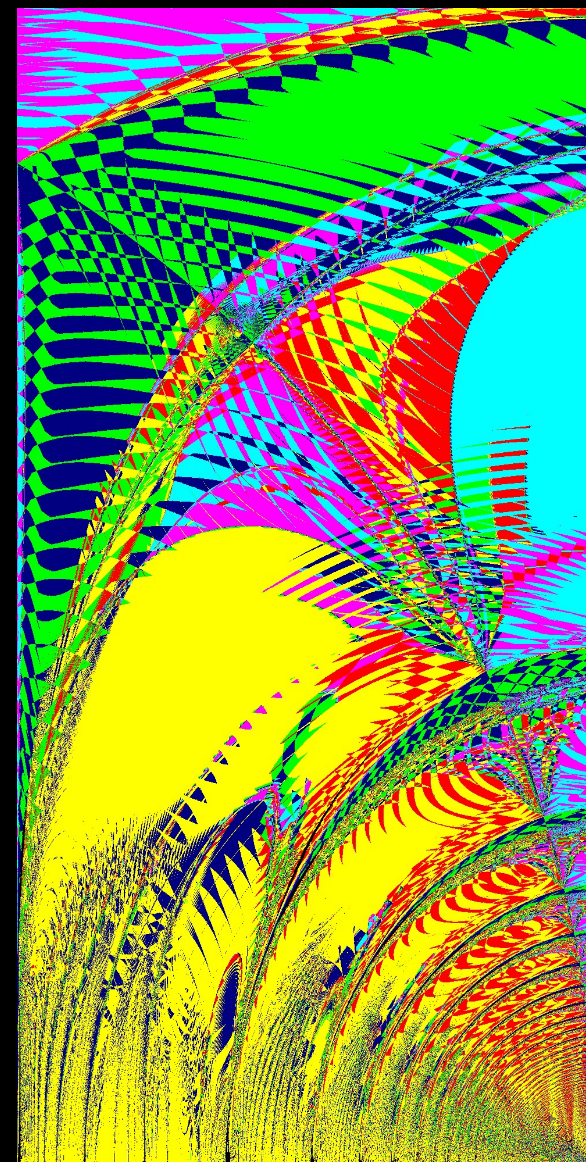

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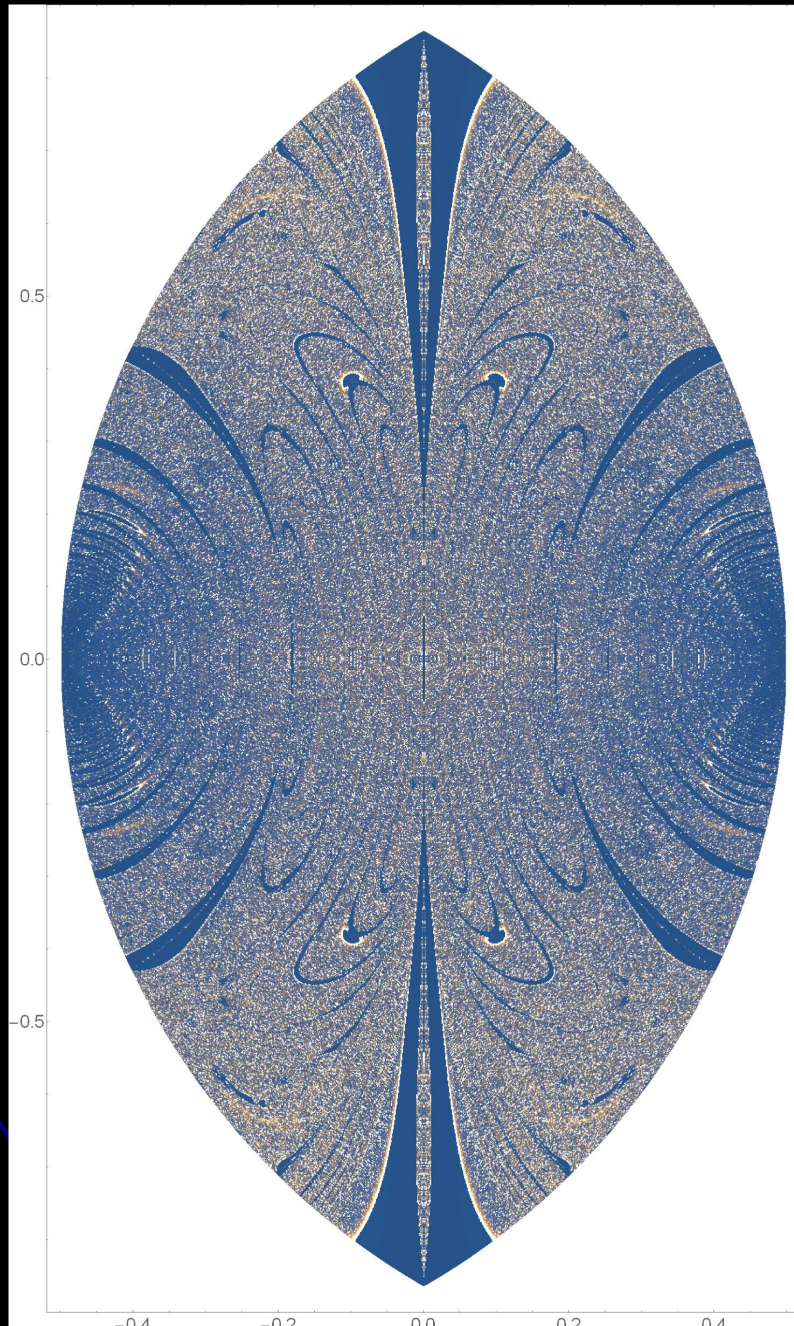
0.5
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0.0
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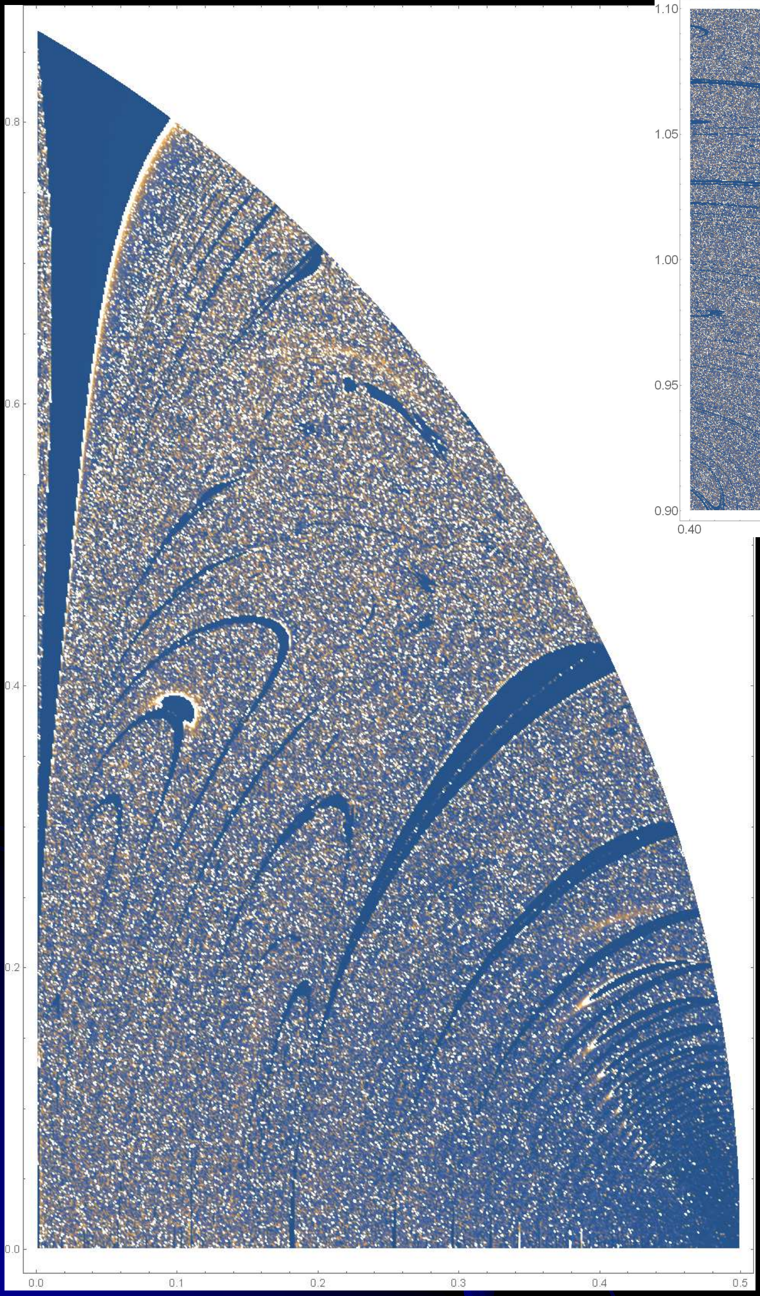
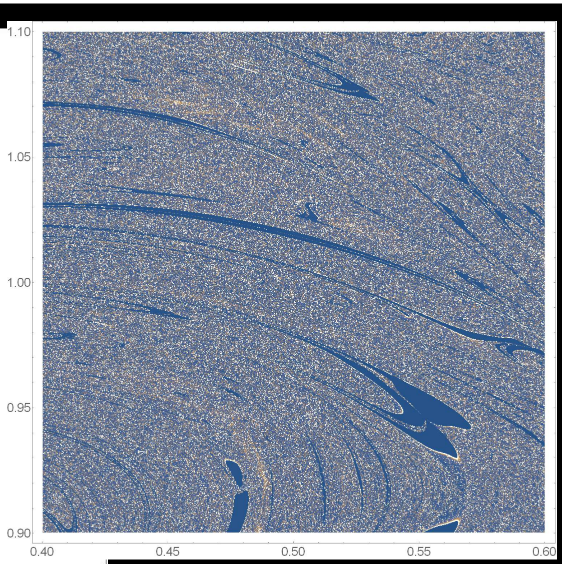
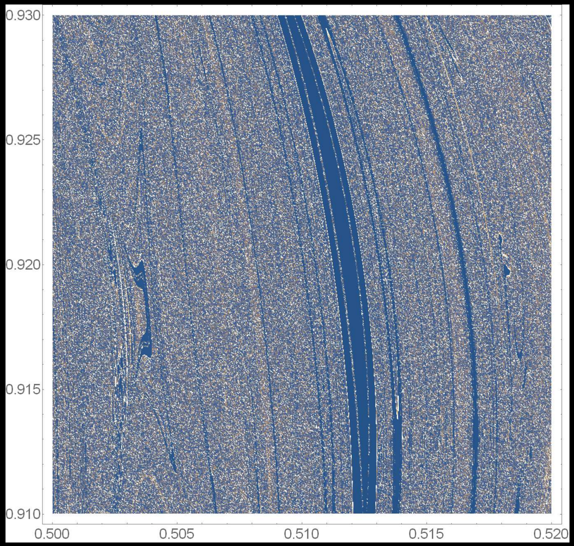
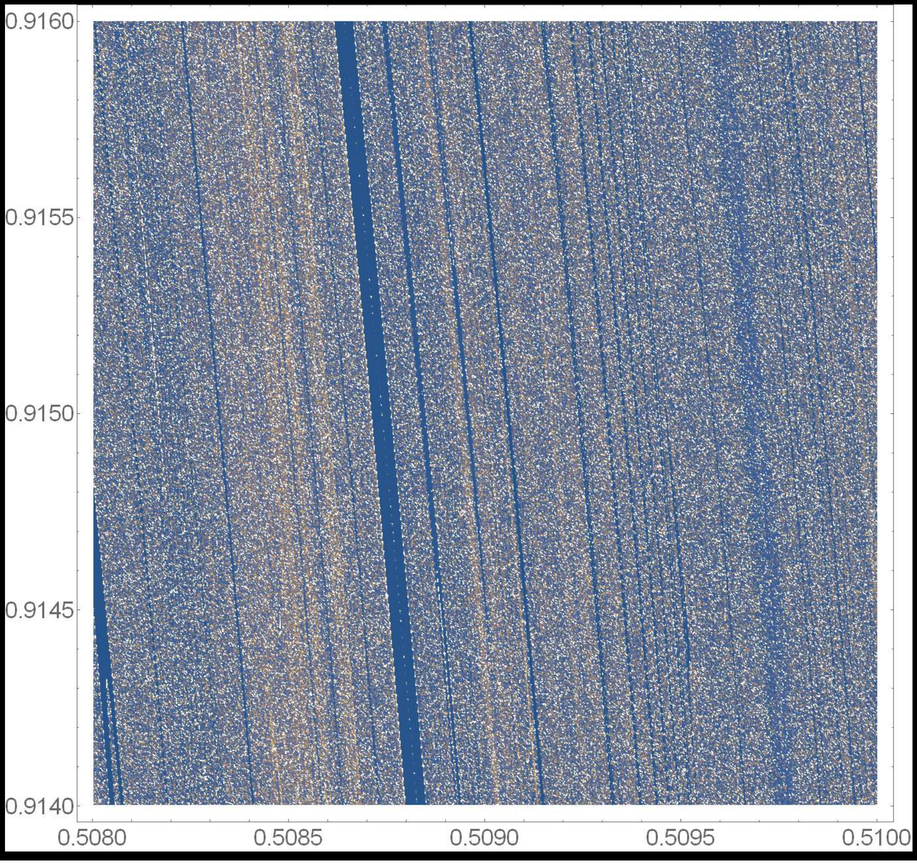
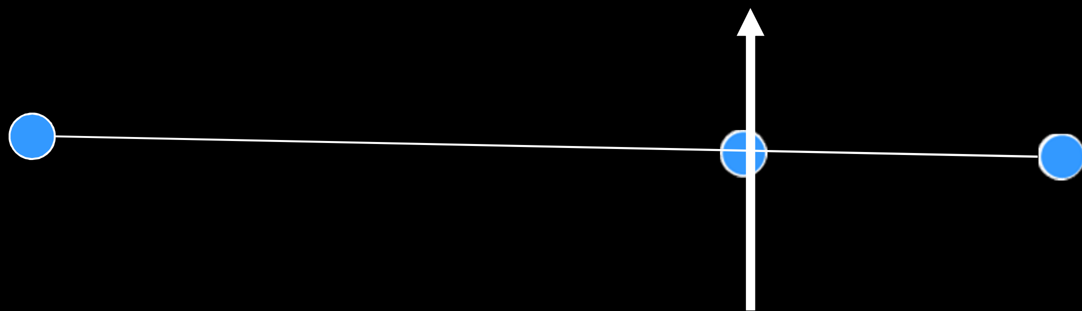




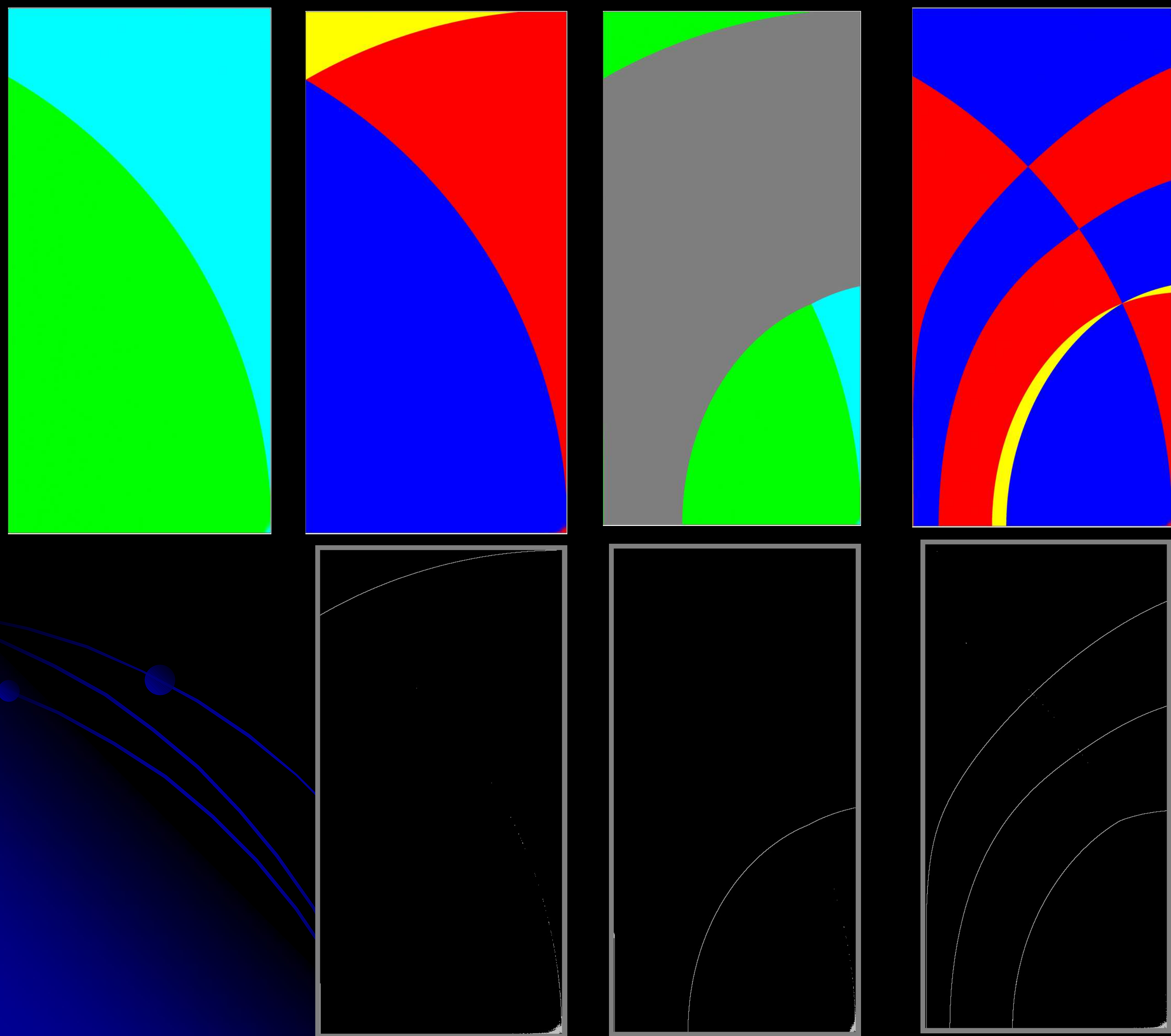


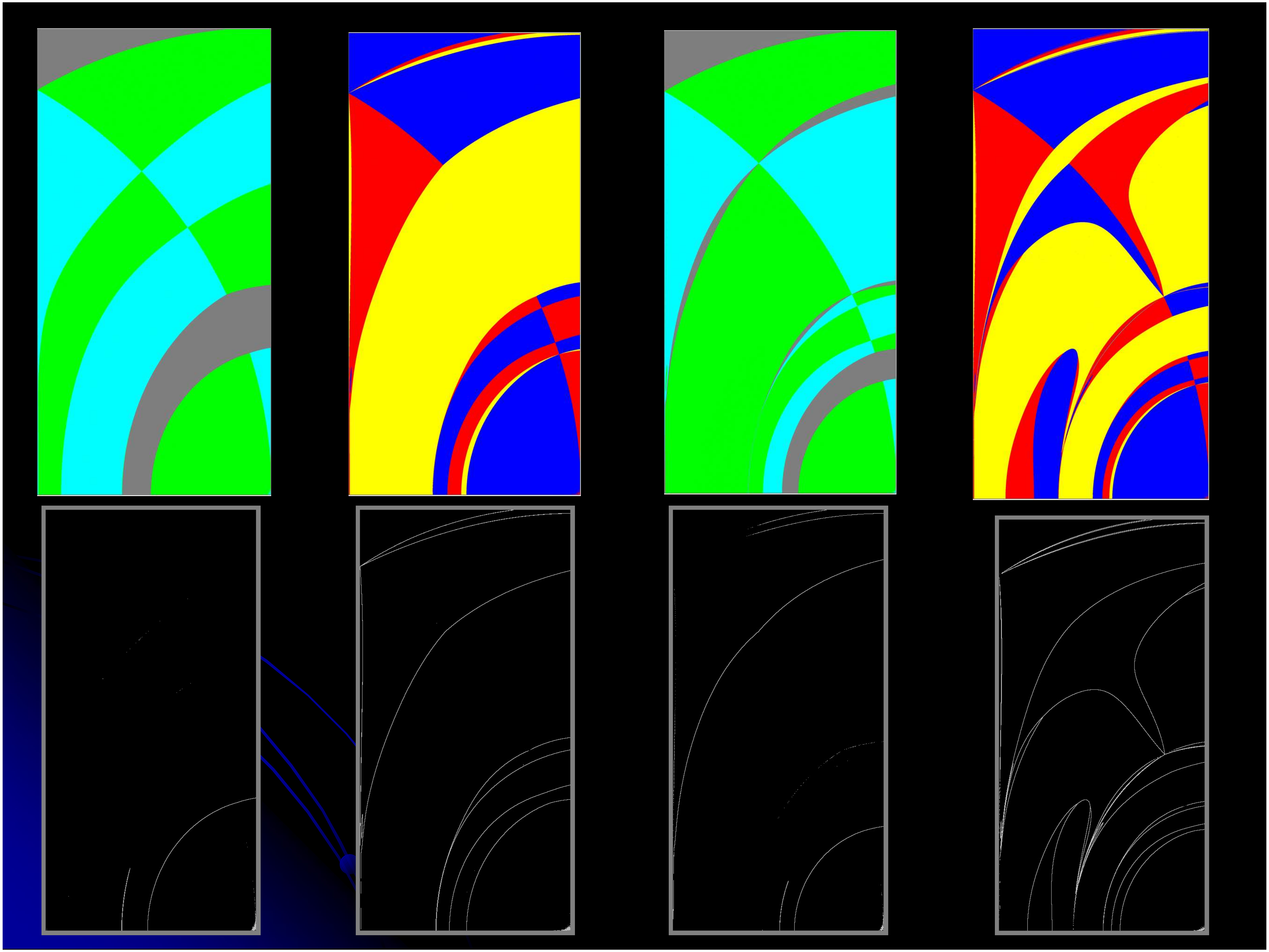


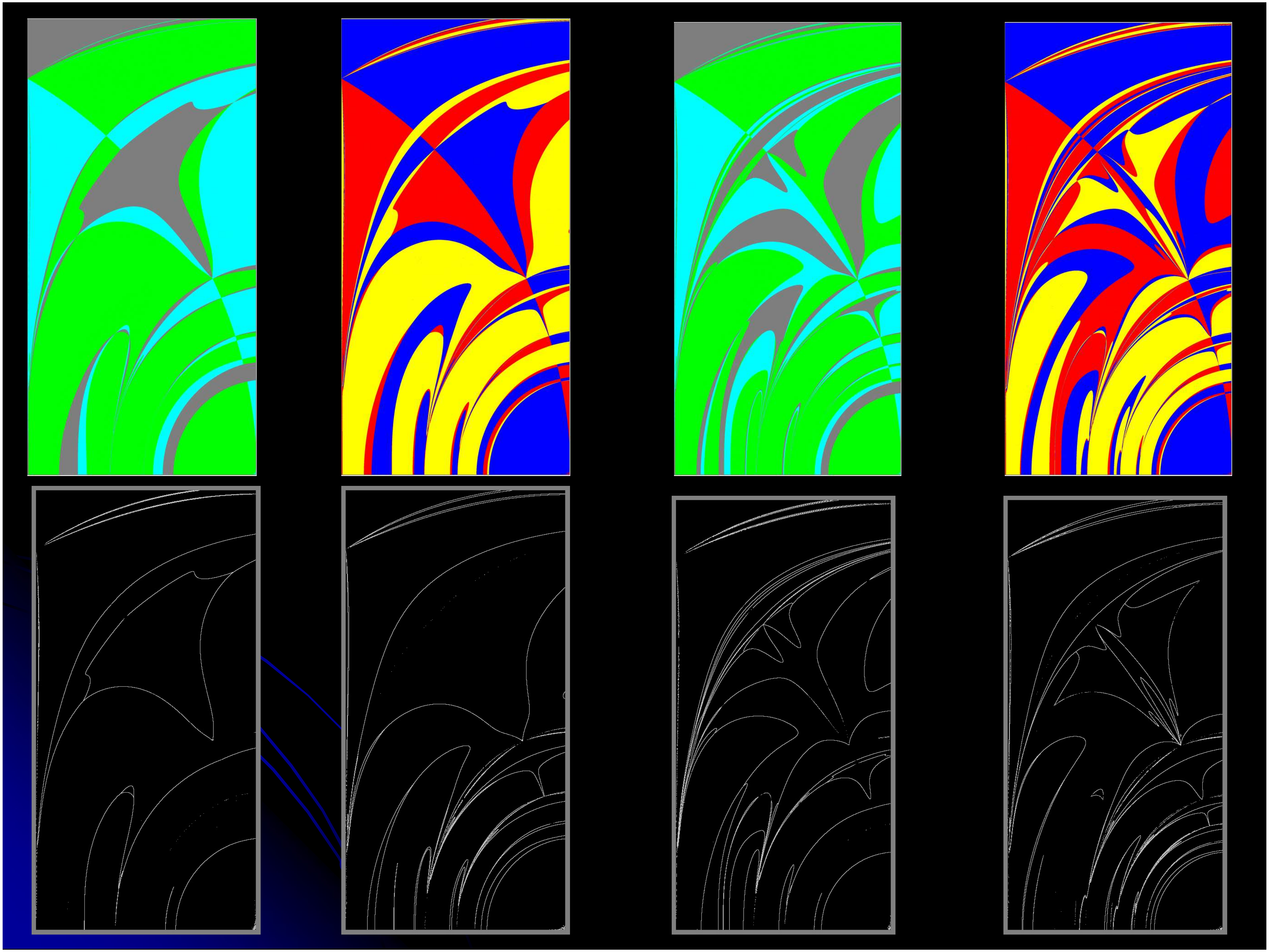
Blue -- short-lived, yellow -- long-lived

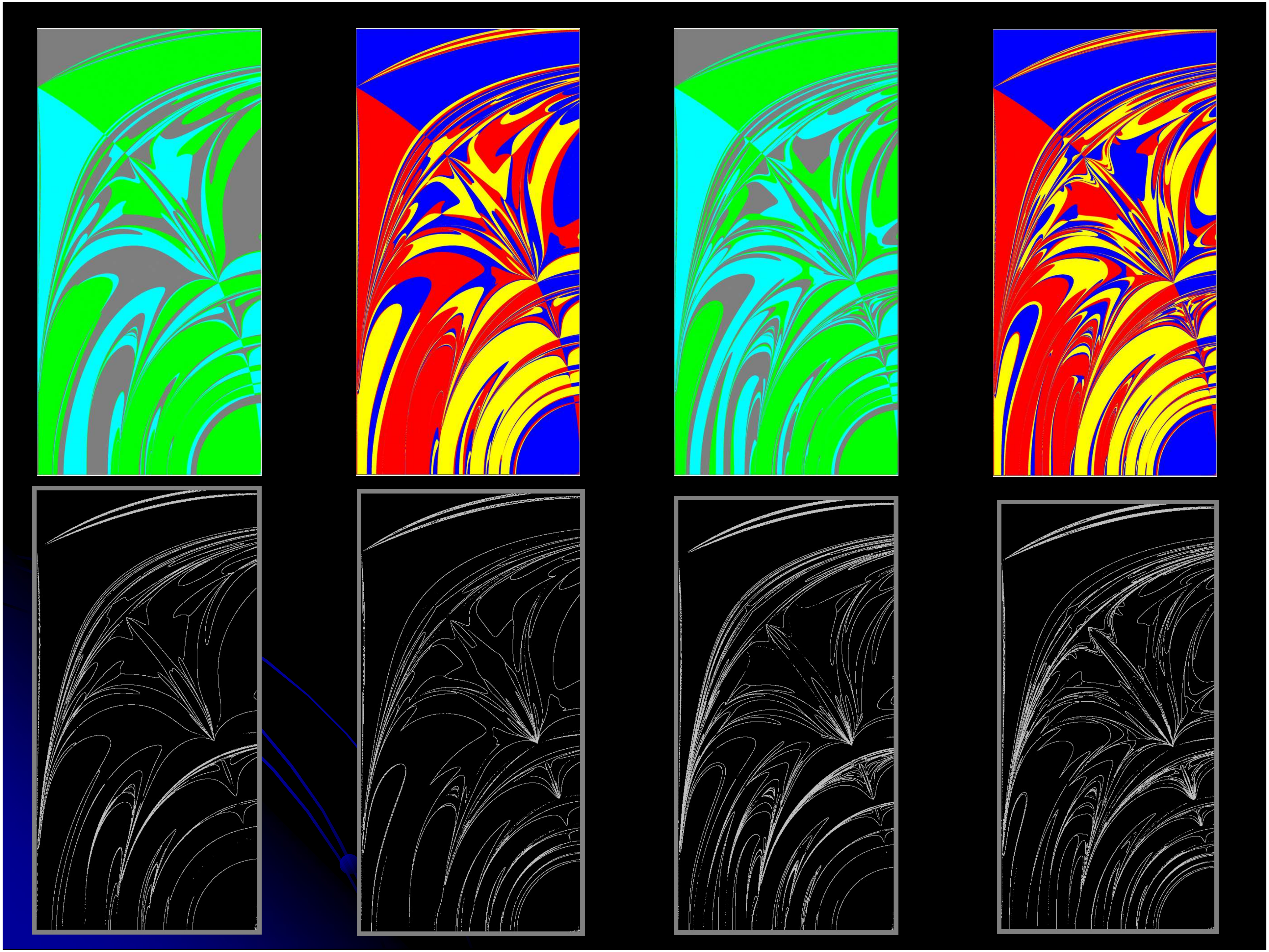


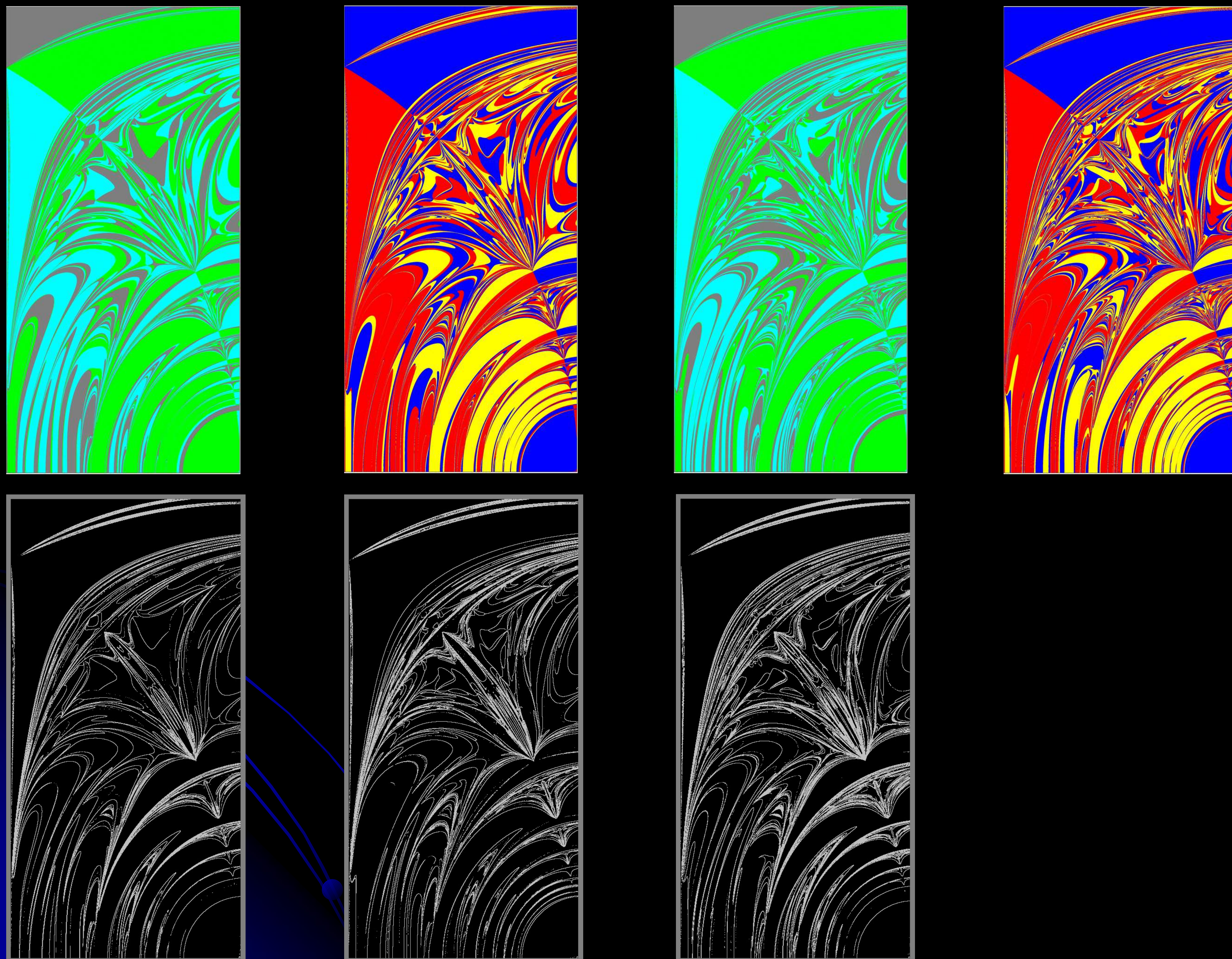
Fixing special dynamical states during the evolution of the triple system (Tanikawa et al.)



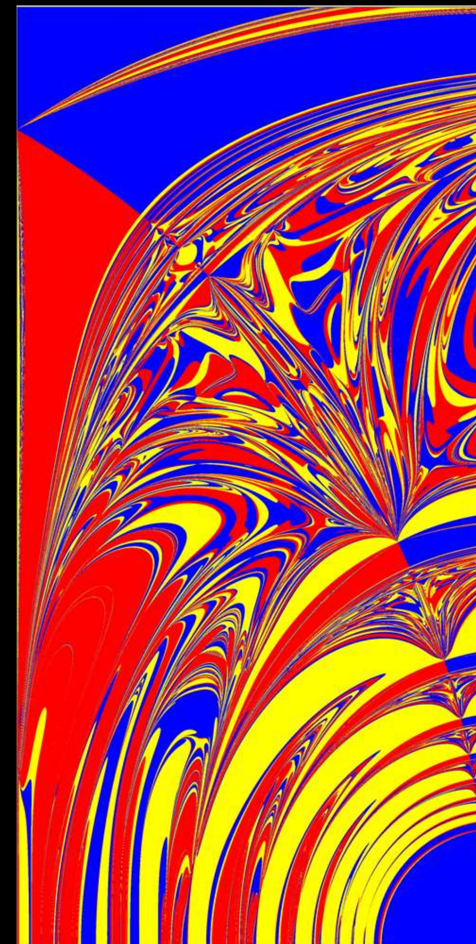


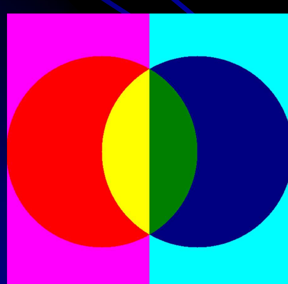


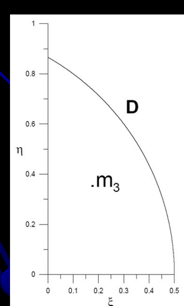


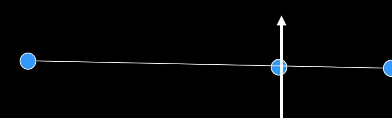


One-to-One Correspondence between Dynamical System and Symbolic Sequence









Entropies to Describe the Complexity of Symbolic Sequences

(Shannon) Entropy

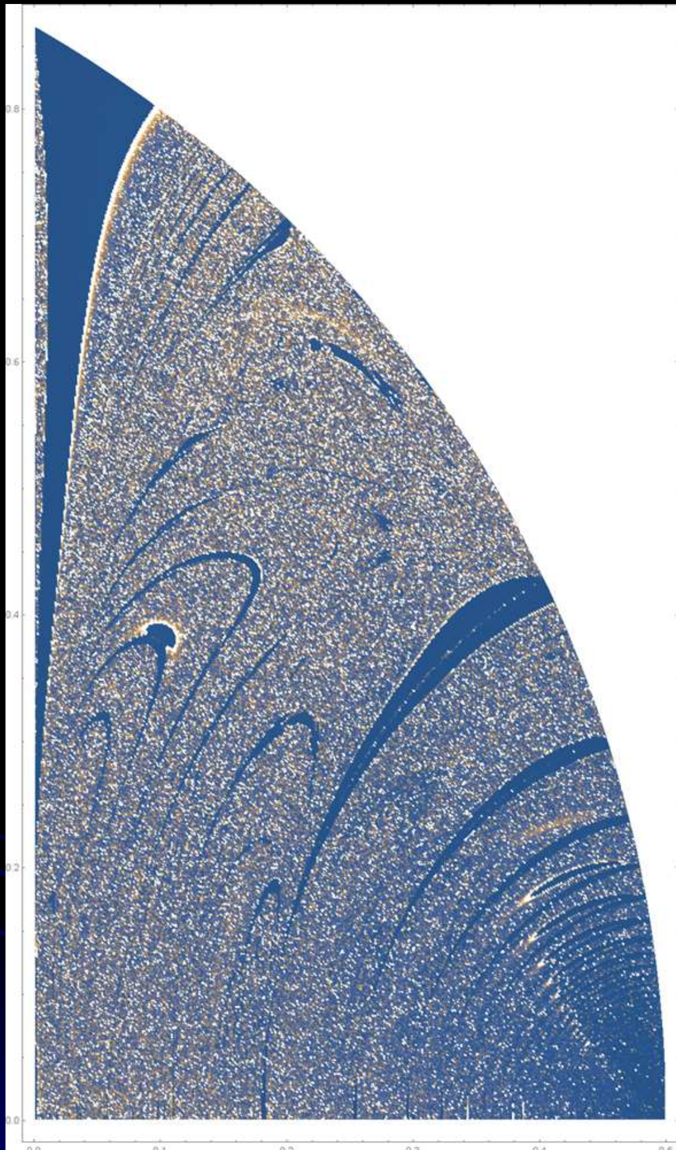
$$H_1 = -\sum_i p_i \ln p_i$$

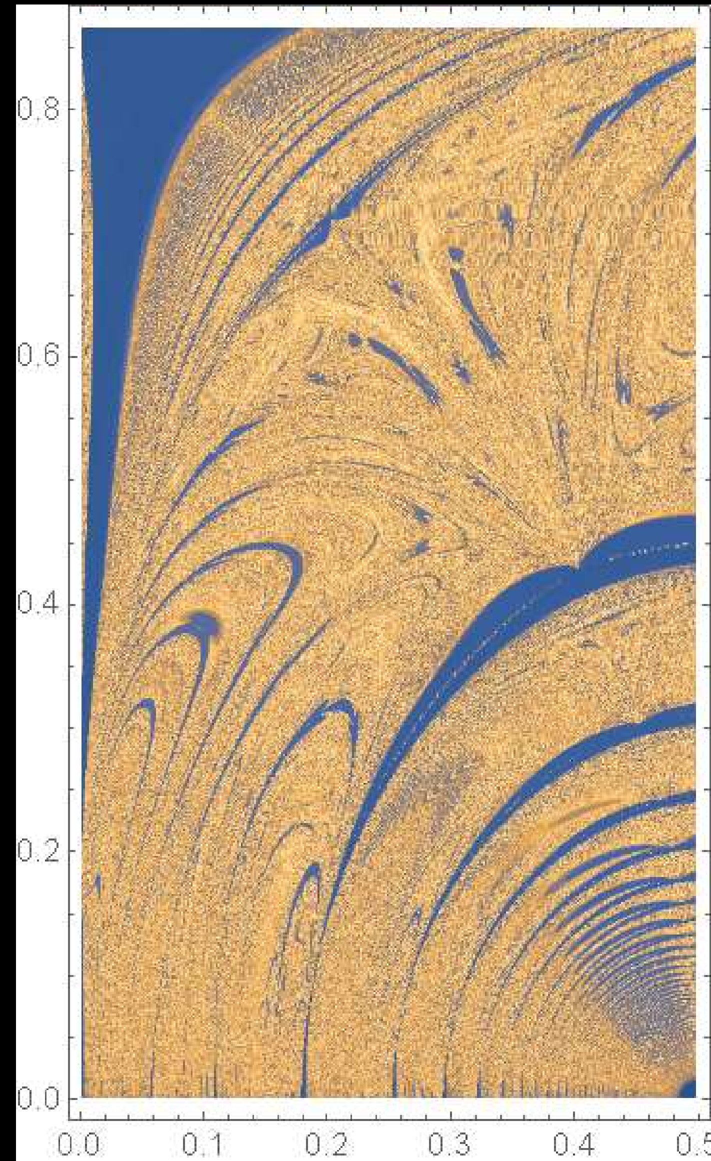
Markov Entropy

$$H_2 = -\sum_i p_i \sum_j q_{ij} \ln q_{ij}$$

p_i – **frequency of symbol *"i"* in the sequence;**

q_{ij} – **frequency of transitions from *"i" to "j"*.**





Values of the (Shannon) entropy in different parts of the Agekian-Anosova map are represented by different colors. Low values are shown in blue; high values are shown in light brown

Methods to Construct Symbolic Sequences

1) Binary encounters

ω_l – number of the distant component;

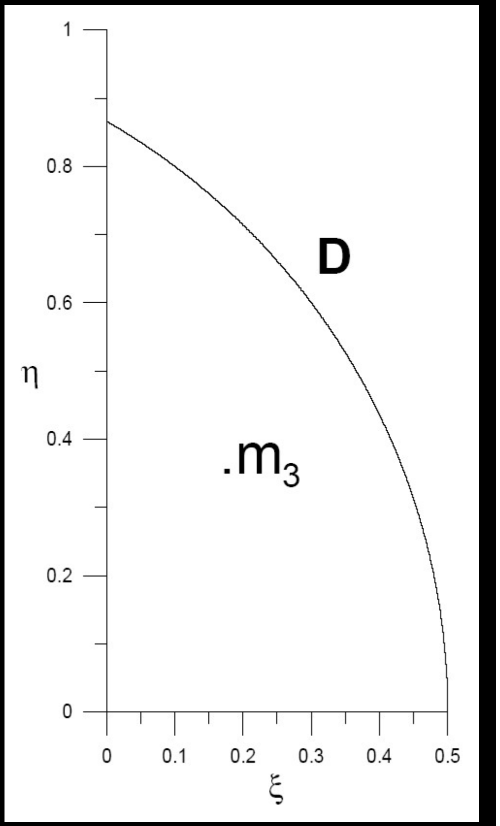
2) Triple encounters

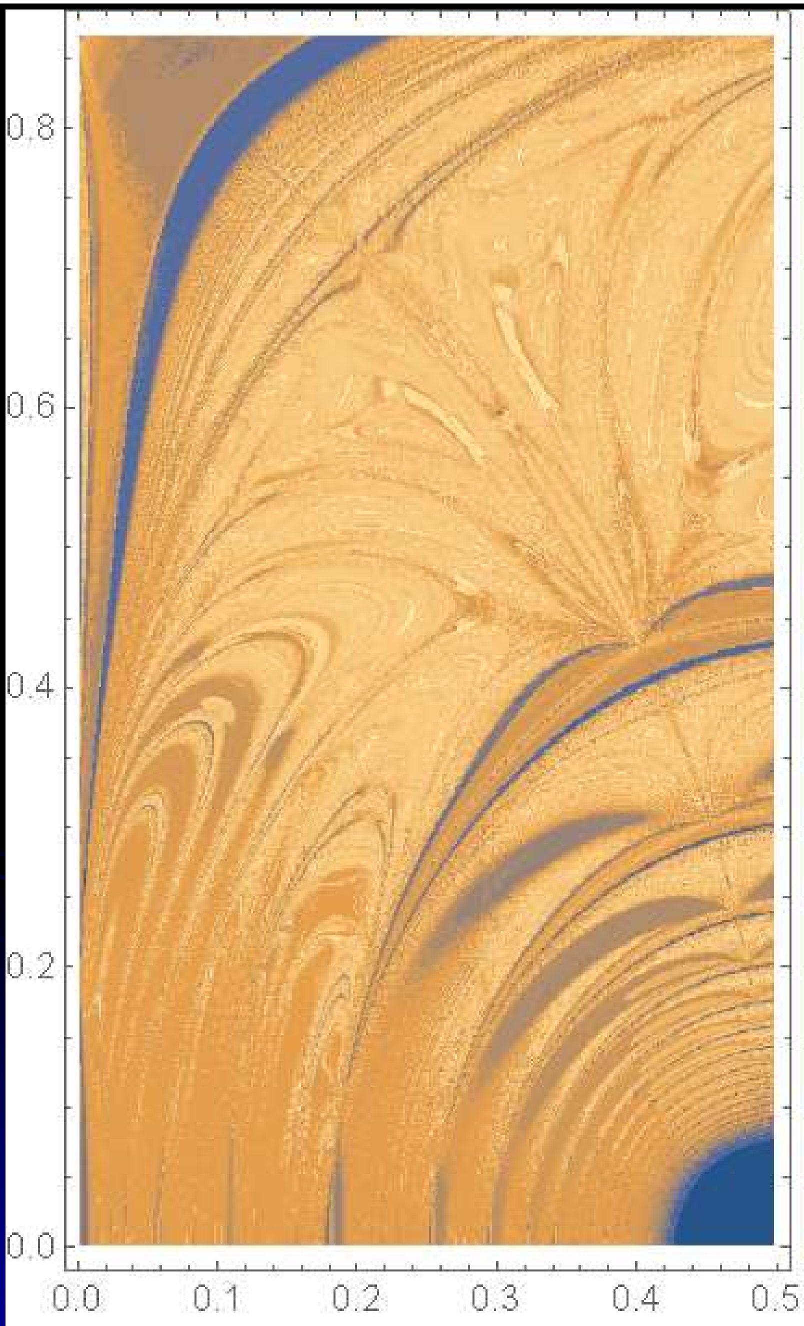
ω_l – number of the distant component;

3) Transitions between subregions of the region D

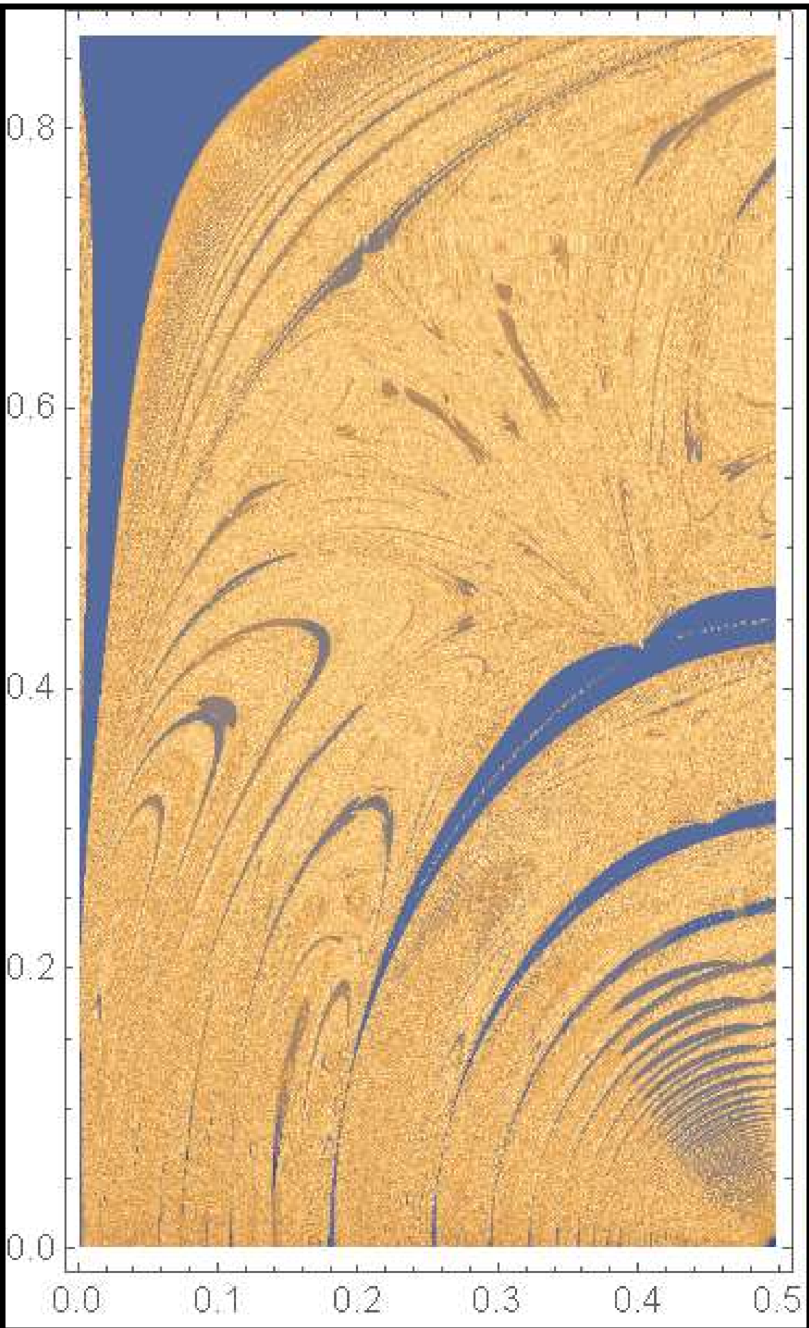
(Agekian & Anosova 1967)

ω_l – number of the subregion.





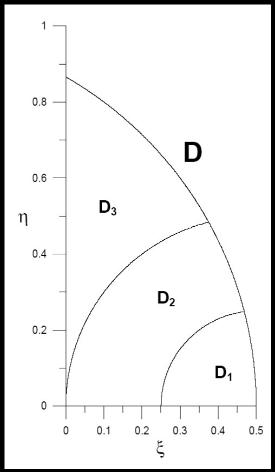
Binary encounters



Triple encounters

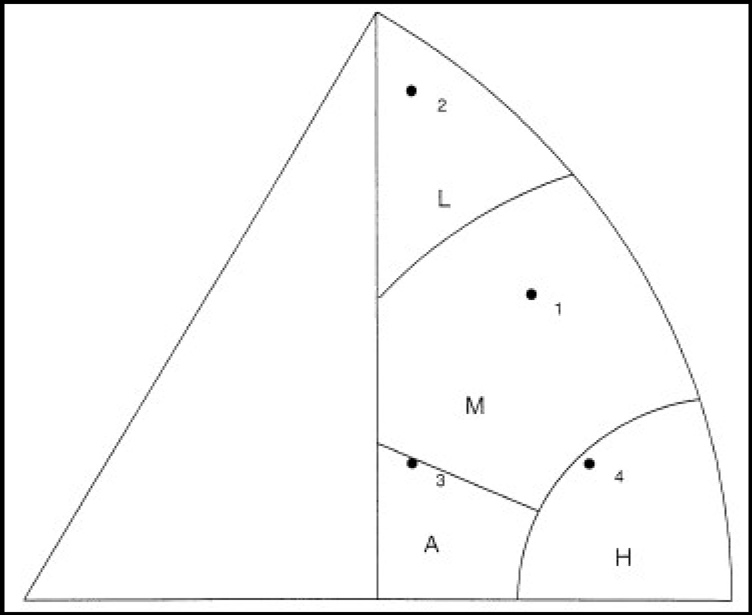
Two Ways to Partition the Region D

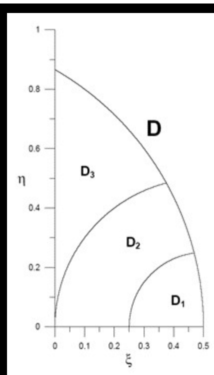
1) Three Subregions



2) Four Subregions

(Chernin et al. 1994)

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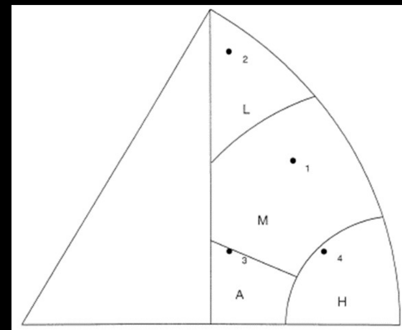
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A
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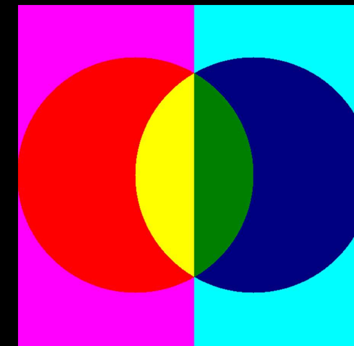

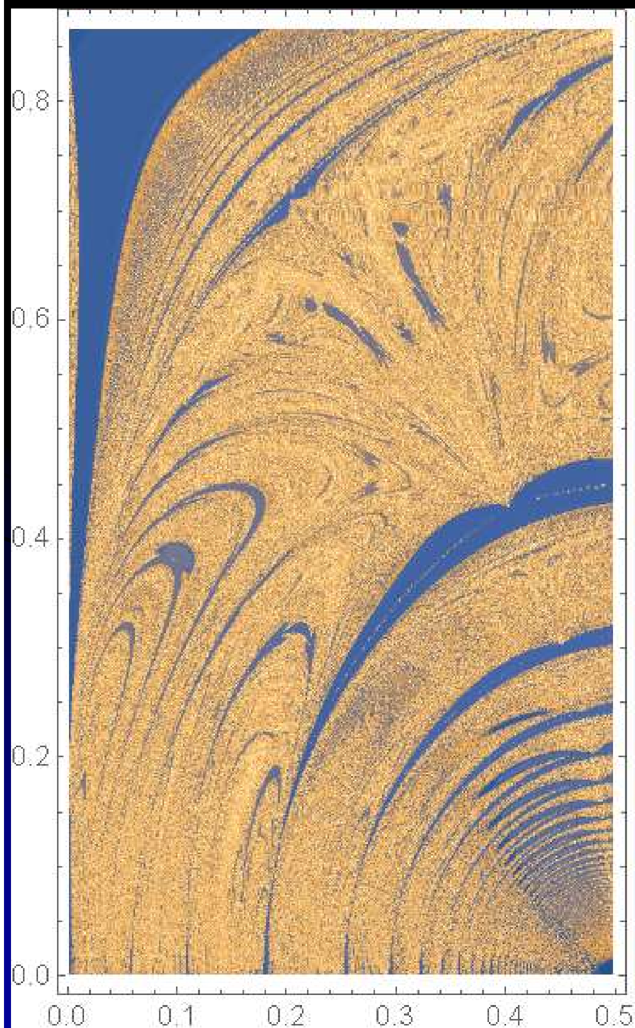
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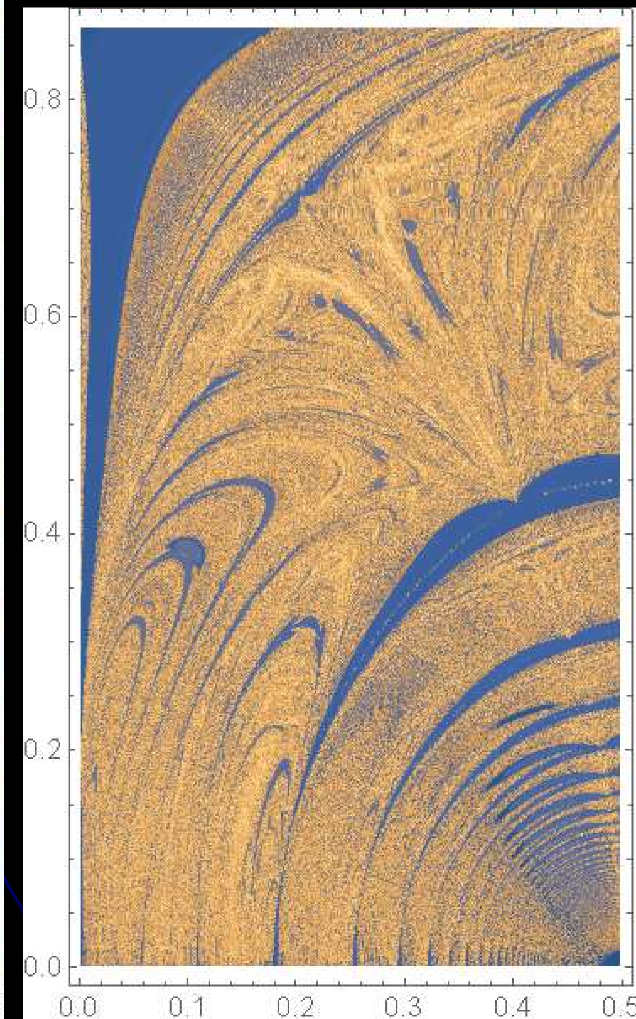
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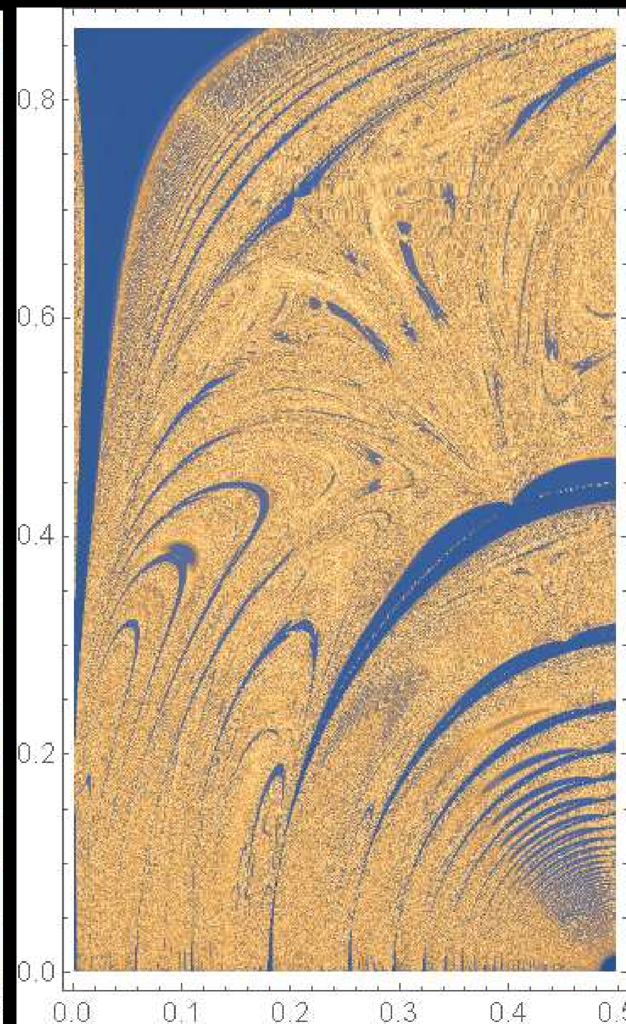
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Entropies as Characteristics of Symbolic Sequences

We study the obtained symbolic sequences:

we estimate the entropies H_1 and H_2 along the trajectory and find their maximum values.

Notations on the Figures for Entropies H_1 and H_2

Lilac color - H_1 , $H_2 \in [0, 0.2]$;

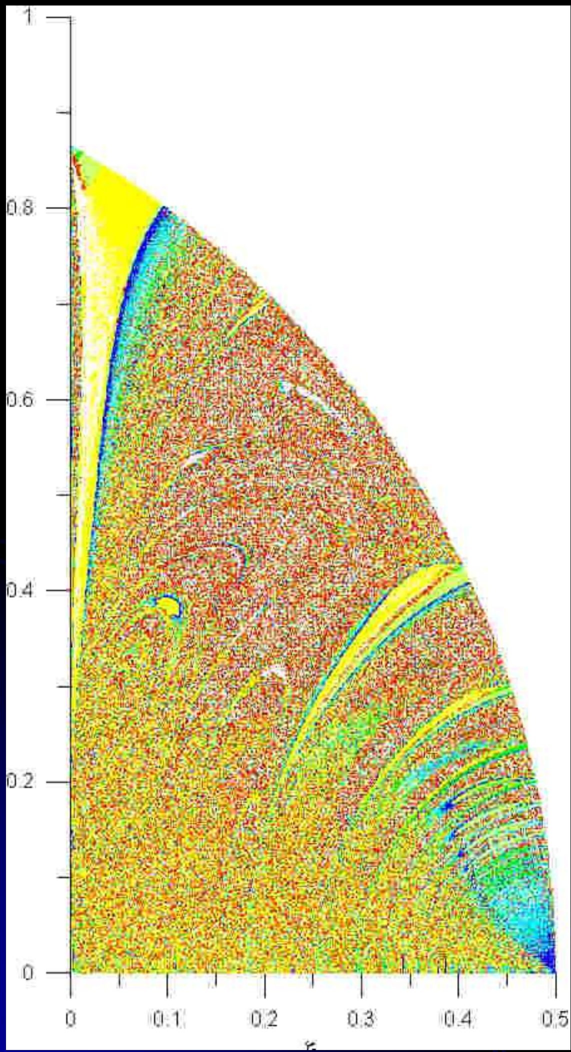
Light blue color - H_1 , $H_2 \in [0.2, 0.4]$;

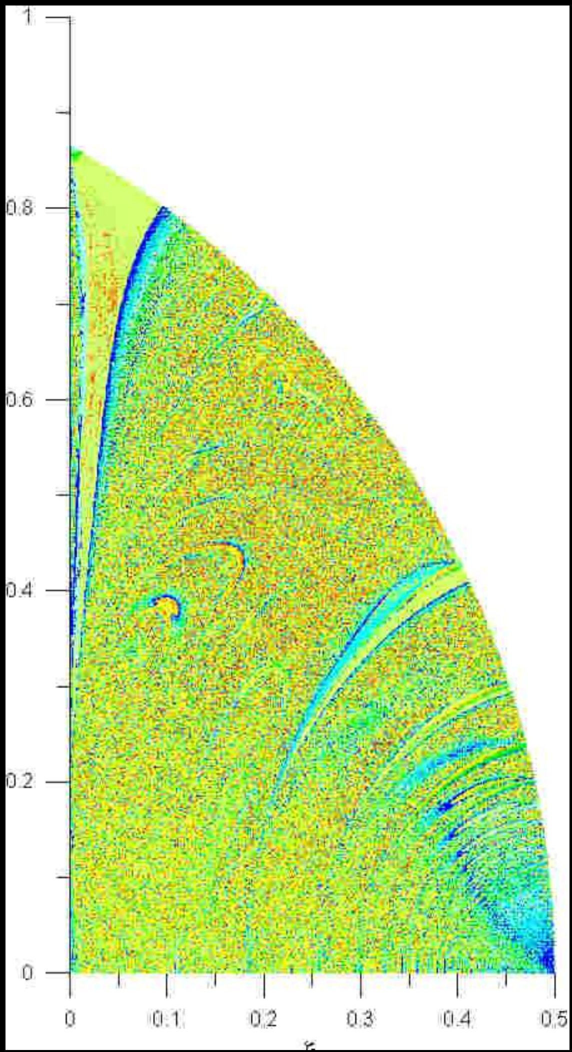
Green color - H_1 , $H_2 \in [0.4, 0.6]$;

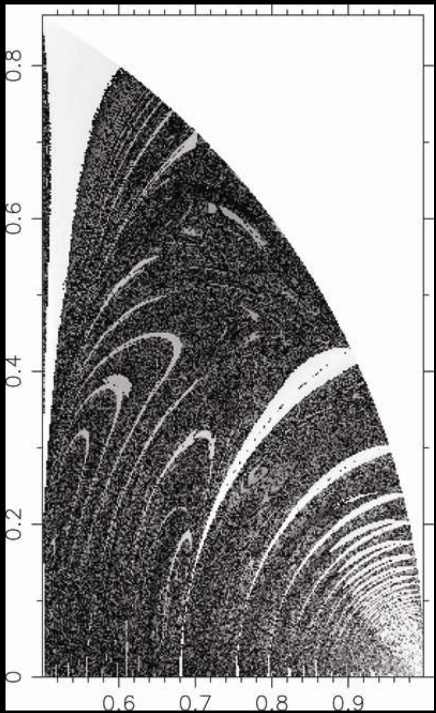
Yellow color - H_1 , $H_2 \in [0.6, 0.8]$;

Red color - H_1 , $H_2 \in [0.8, 1.0]$.

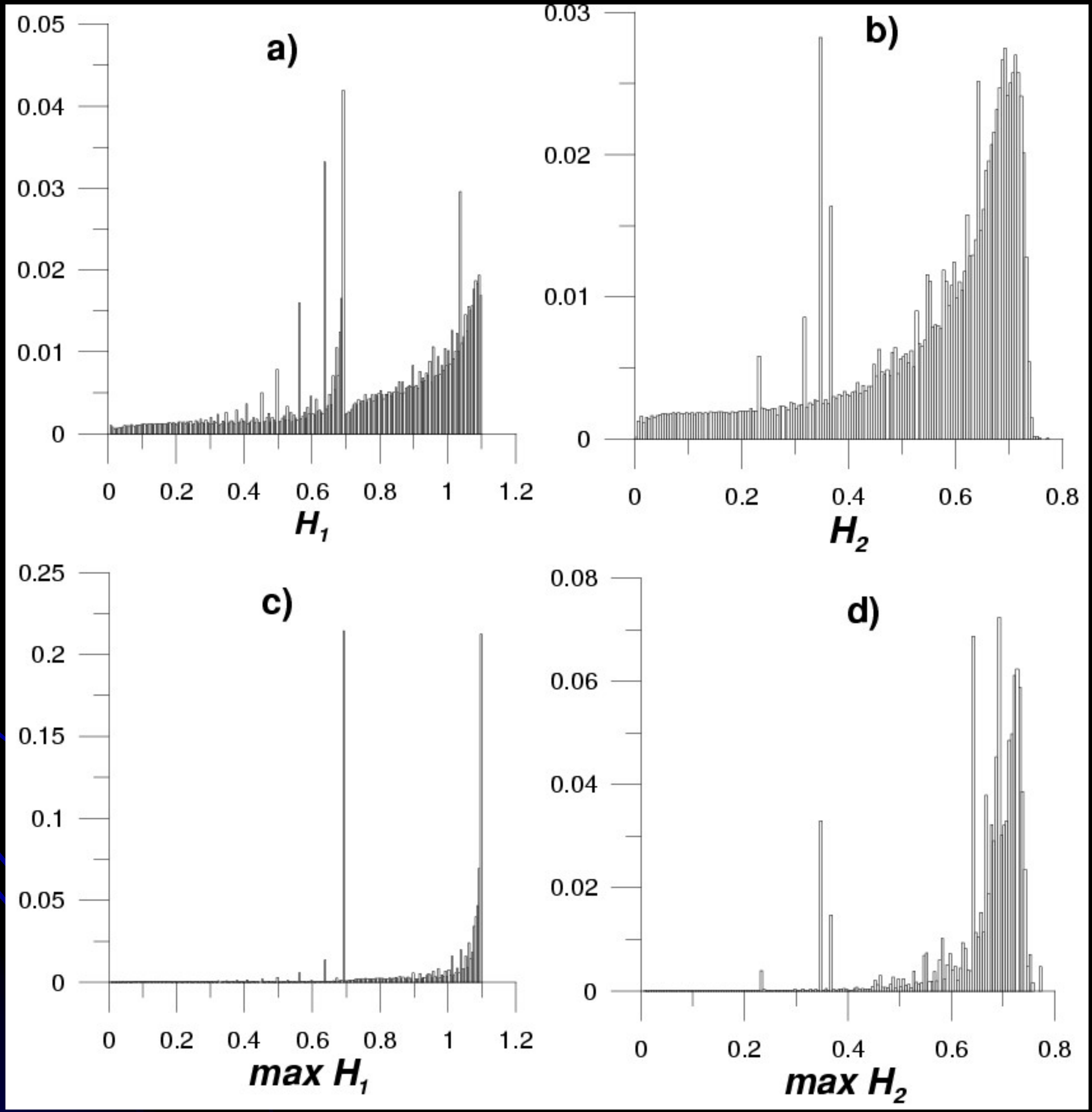
Entropies $H_1(\xi, \eta)$; $H_2(\xi, \eta)$ for double encounters



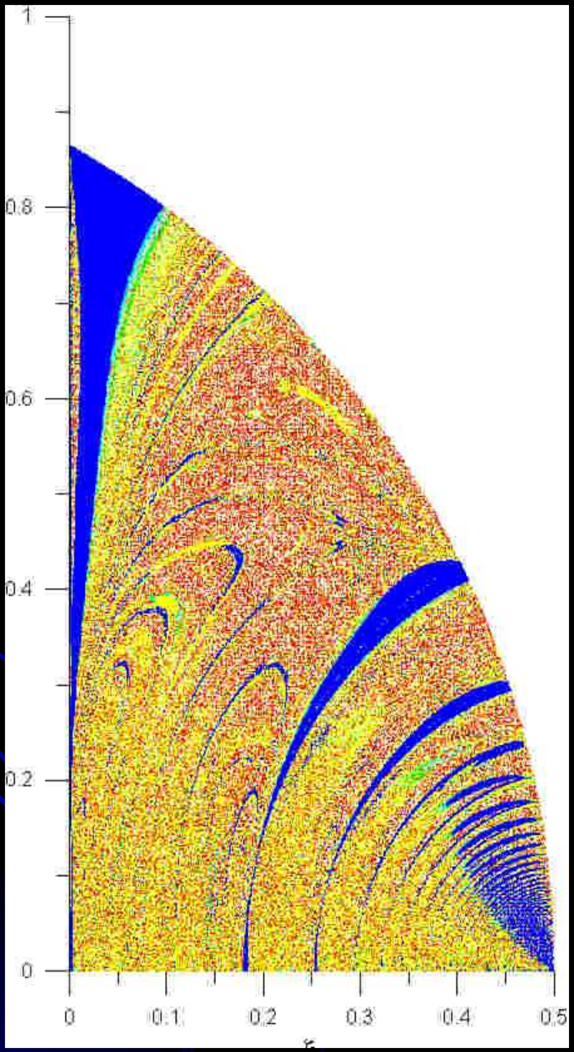


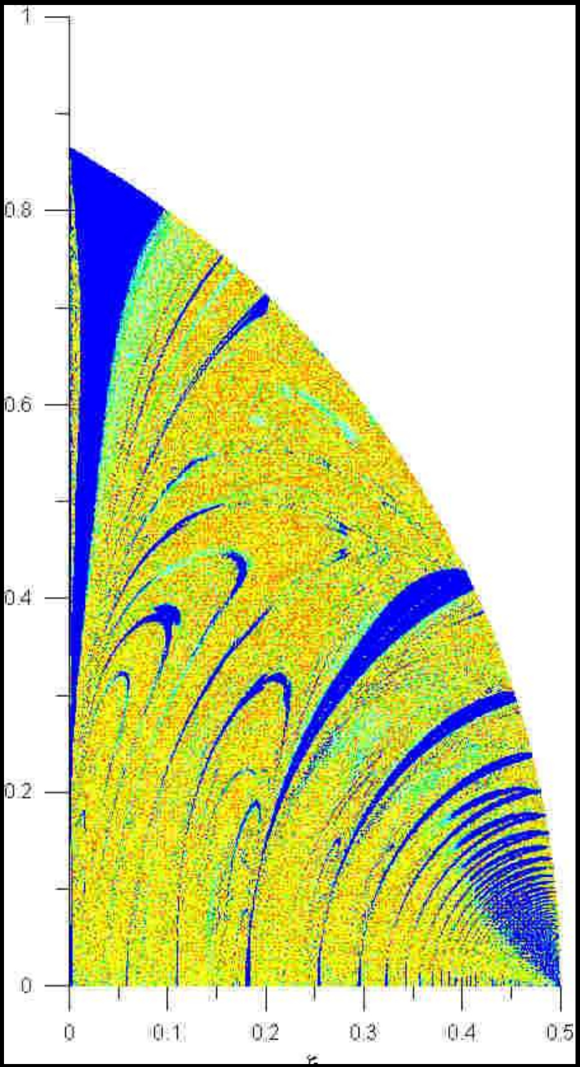


Entropies $H_1(\xi, \eta)$; $H_2(\xi, \eta)$ for double encounters

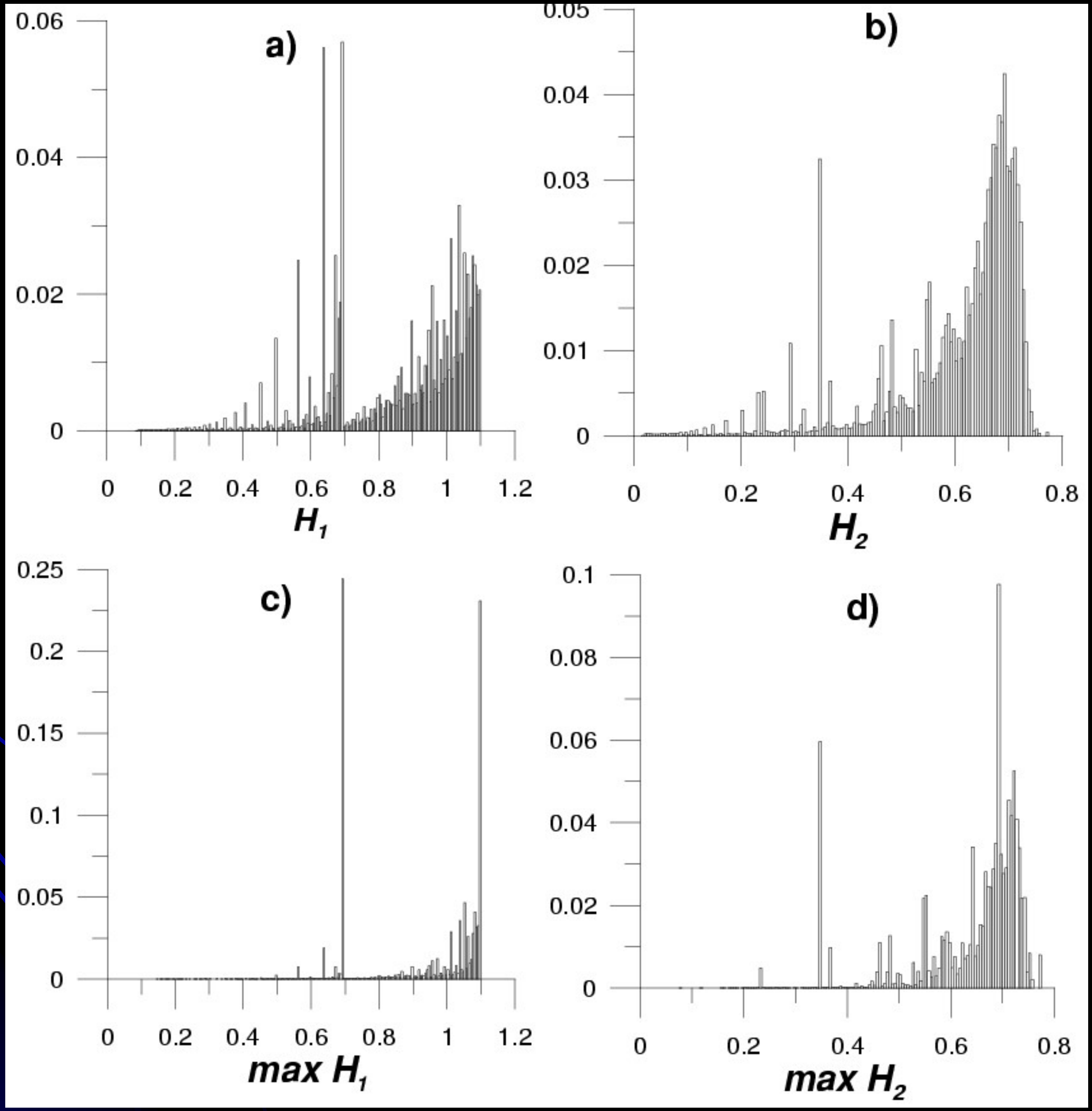


Entropies $H_1(\xi, \eta)$; $H_2(\xi, \eta)$ for triple encounters

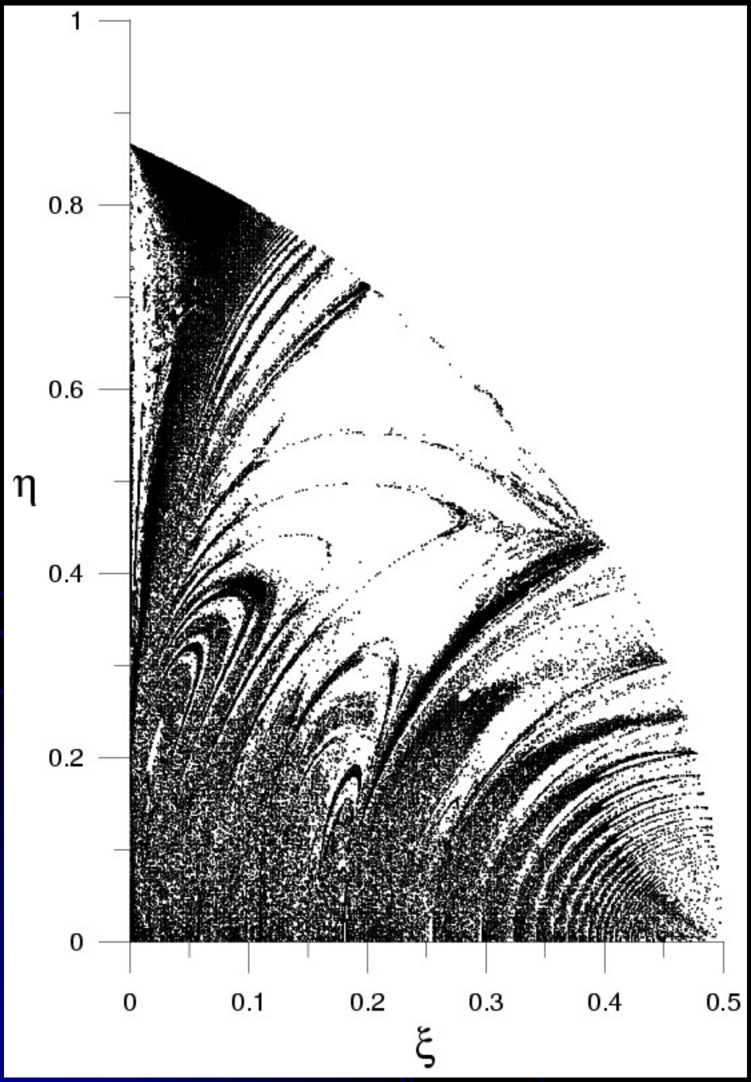




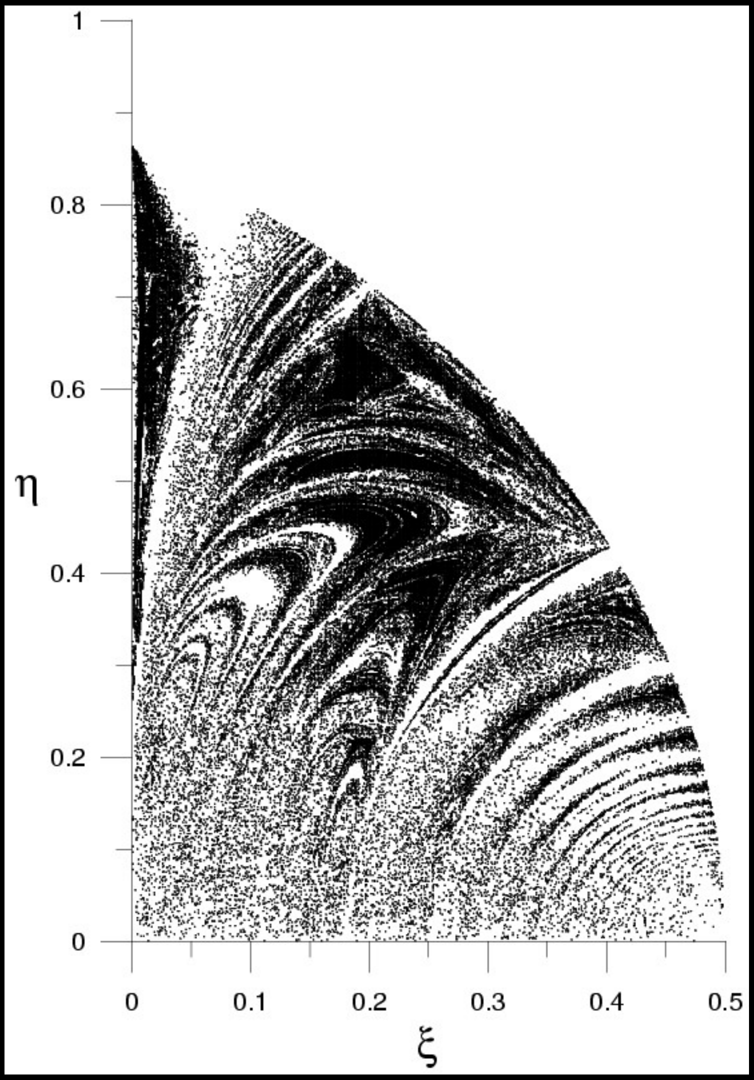
Entropies $H_1(\xi, \eta)$; $H_2(\xi, \eta)$ for triple encounters



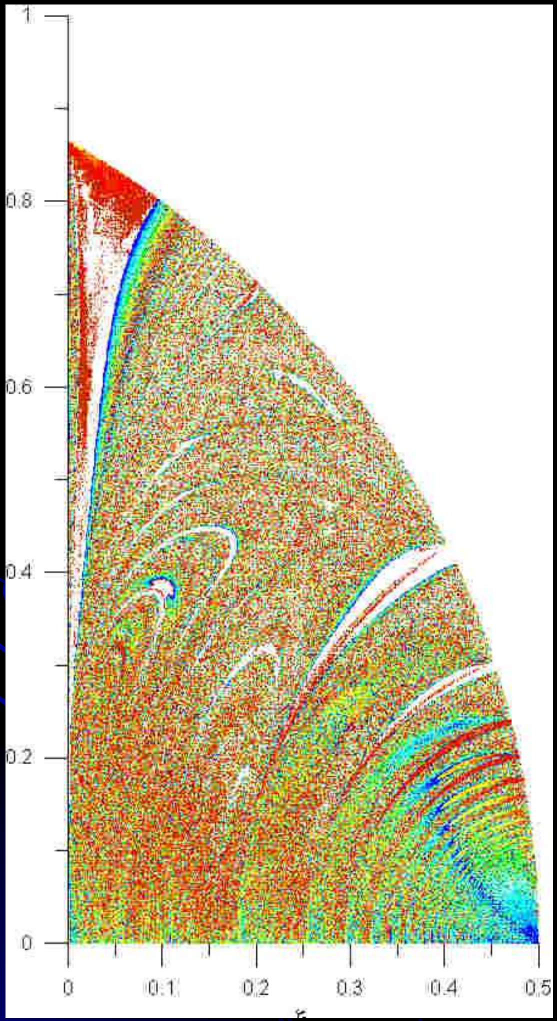
0.69 < H_1max < 0.70

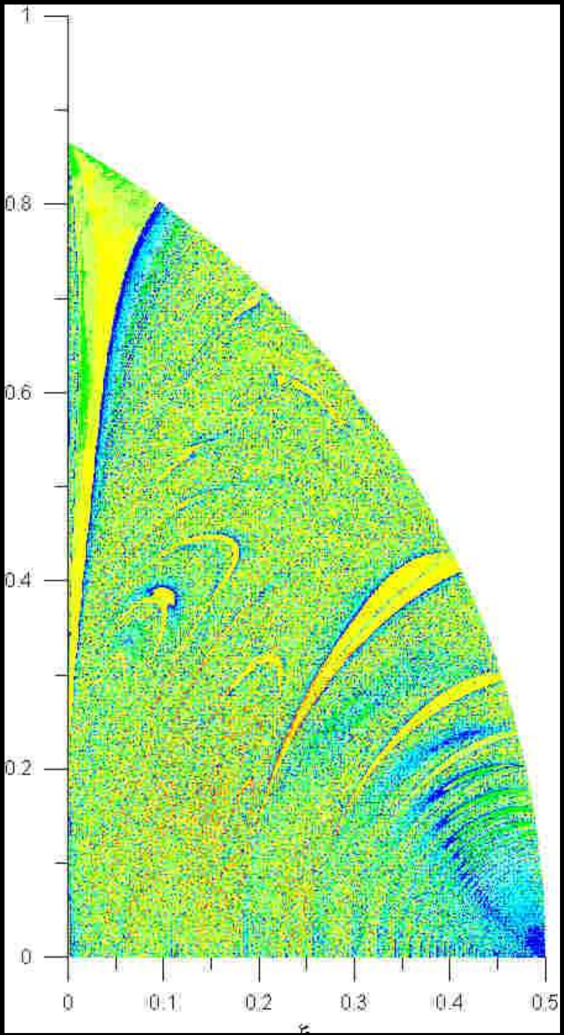


1.09 < H_1max < 1.10

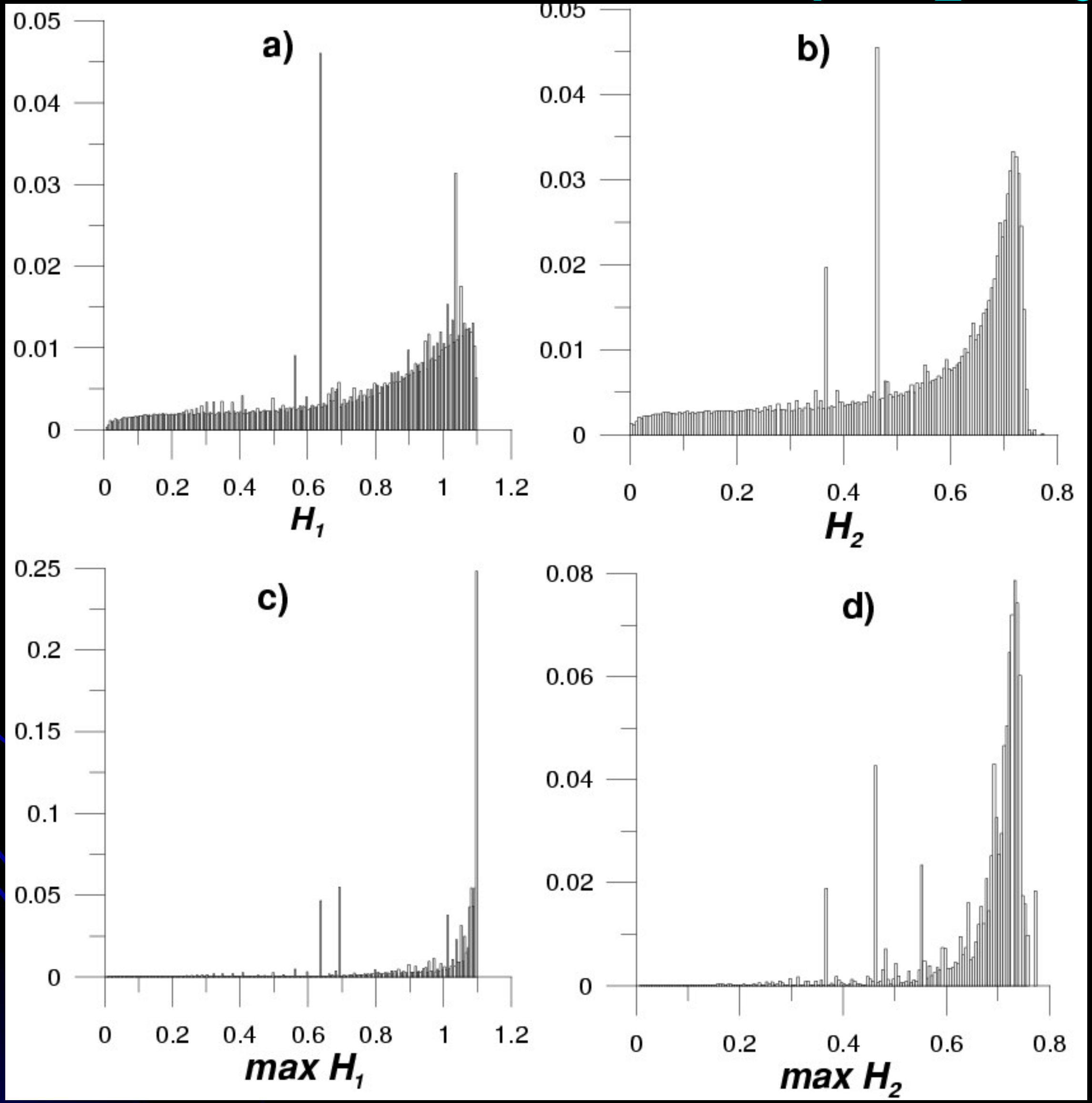


Entropies $H_1(\xi, \eta)$; $H_2(\xi, \eta)$ *for subregions* D_1, D_2, D_3

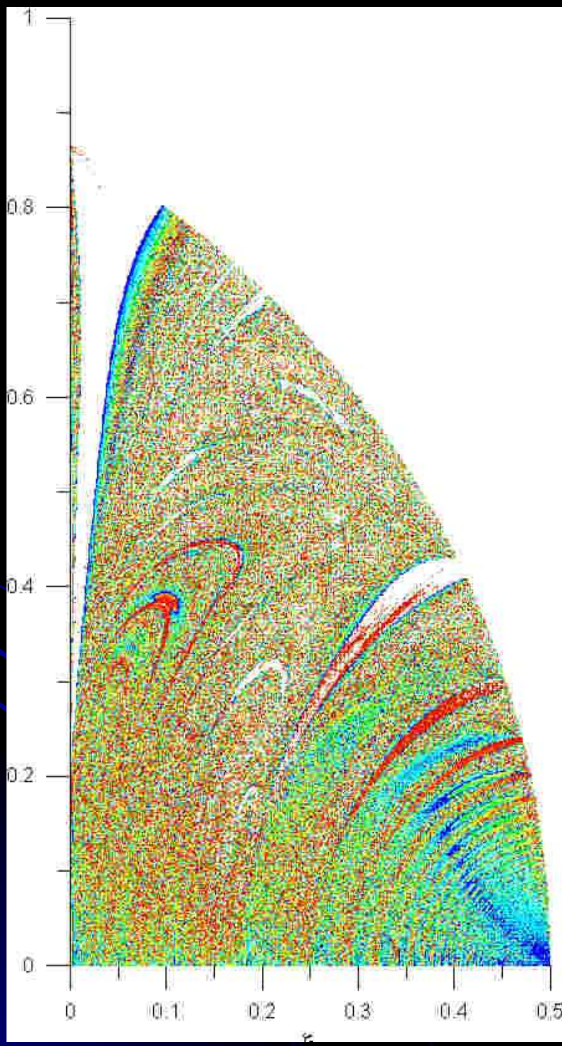


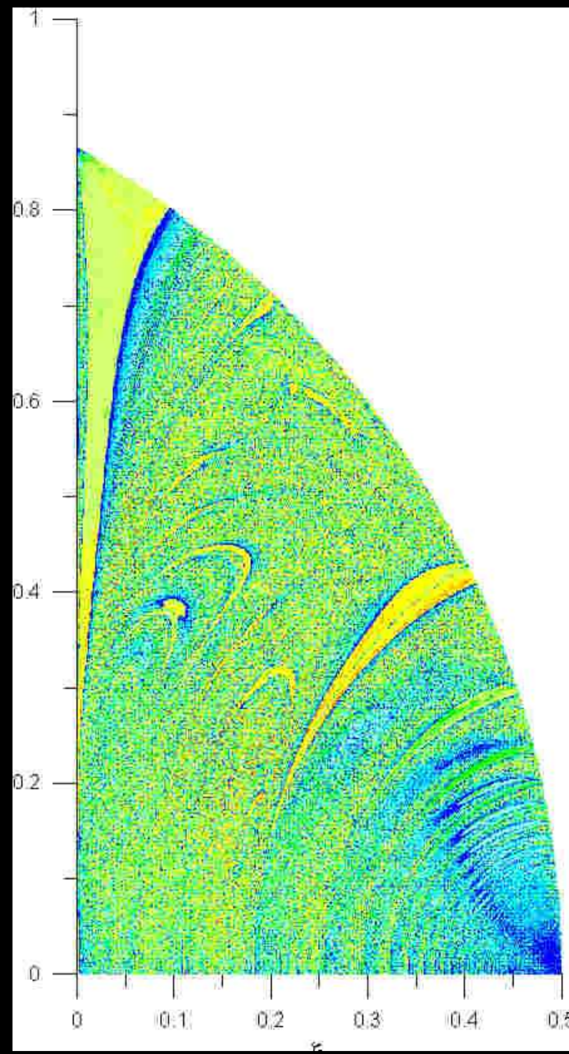


Entropies $H_1(\xi, \eta)$; $H_2(\xi, \eta)$ *for subregions* D_1, D_2, D_3

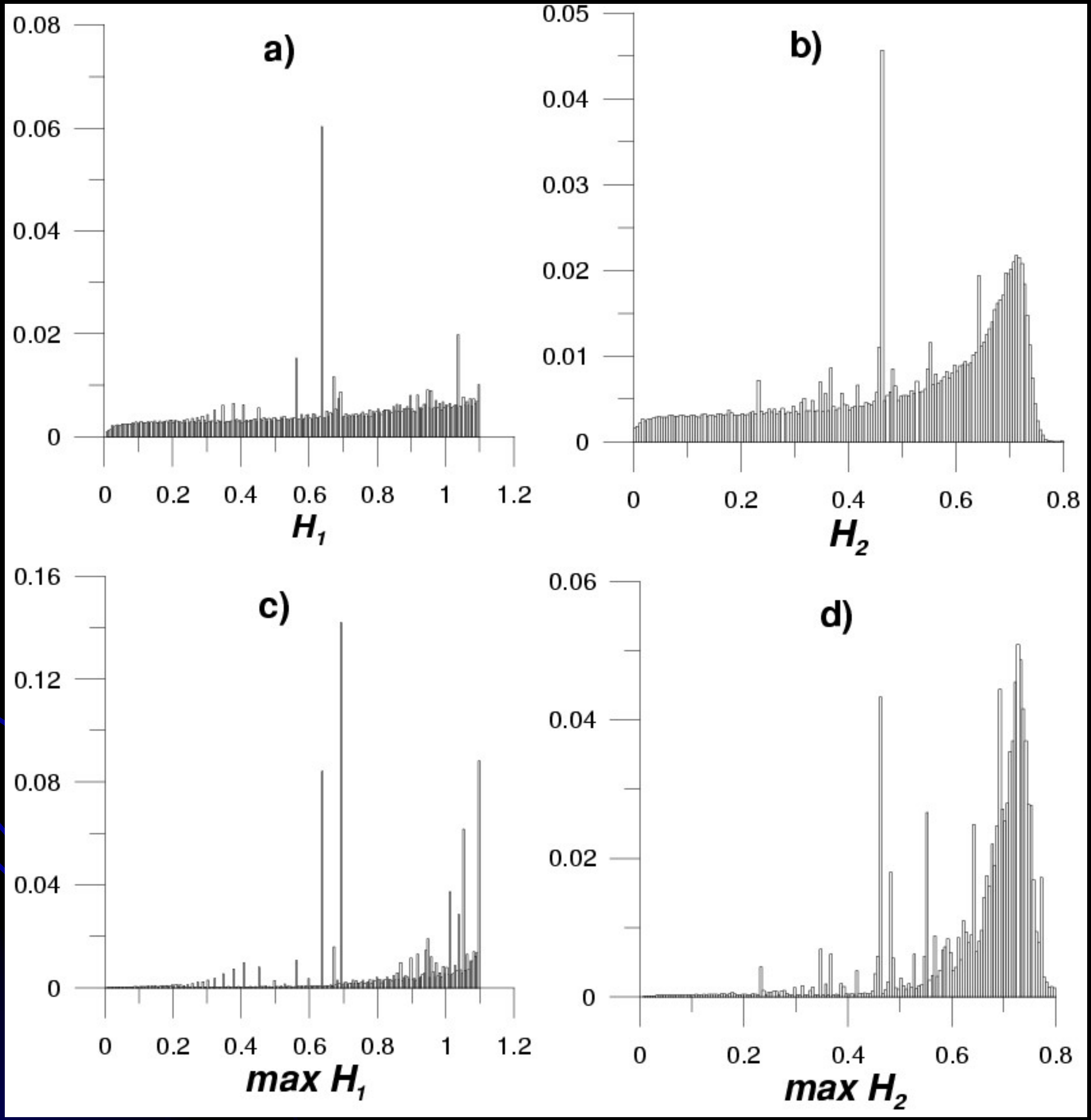


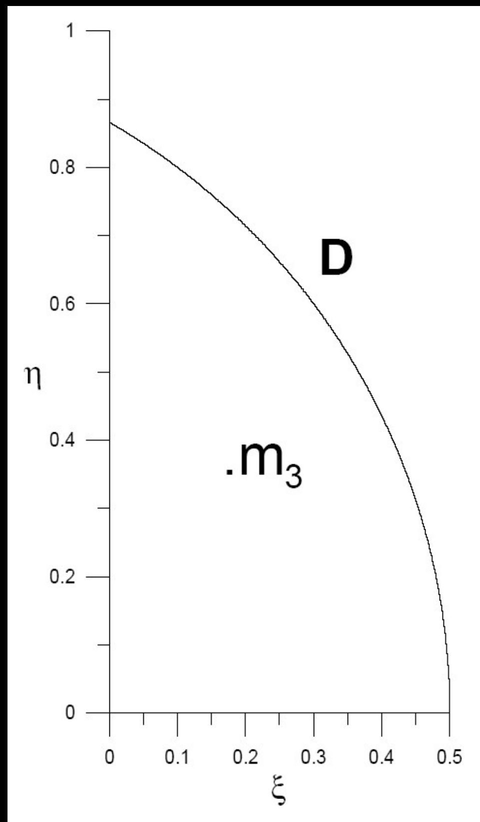
Entropies $H_1(\xi, \eta)$; $H_2(\xi, \eta)$ *for subregions* A, H, L, M





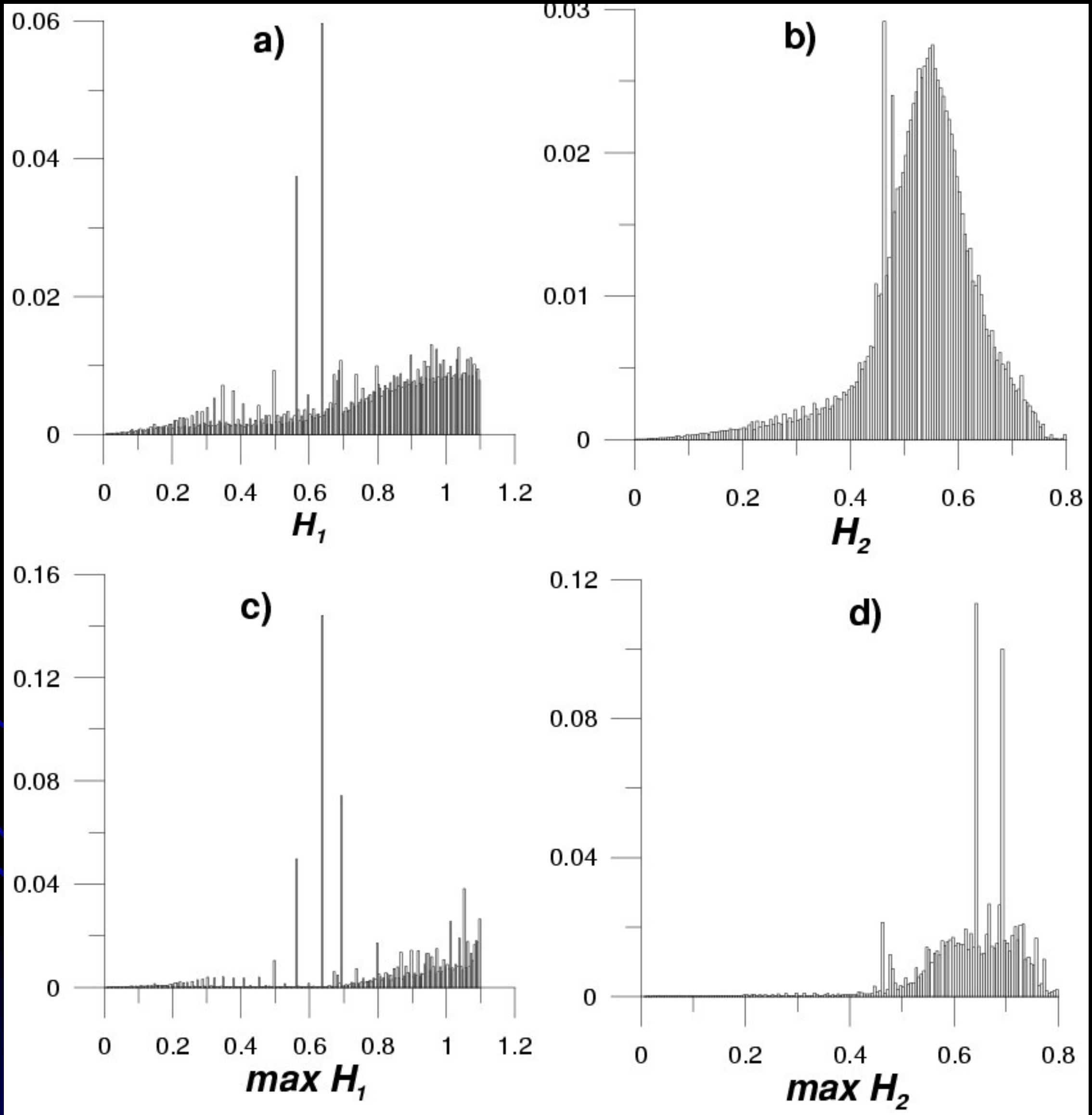
Entropies $H_1(\xi, \eta)$; $H_2(\xi, \eta)$ *for subregions* A, H, L, M



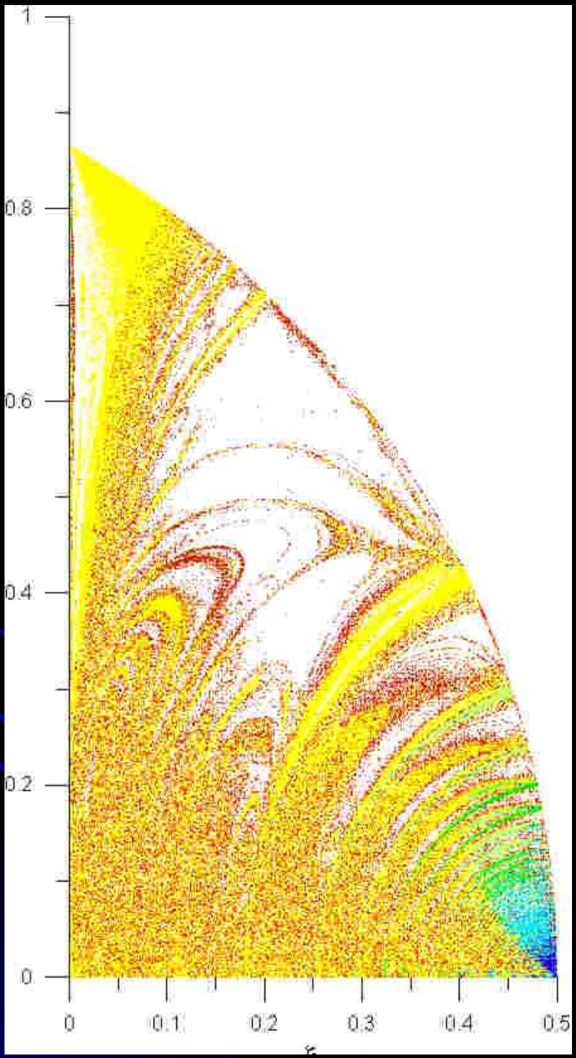


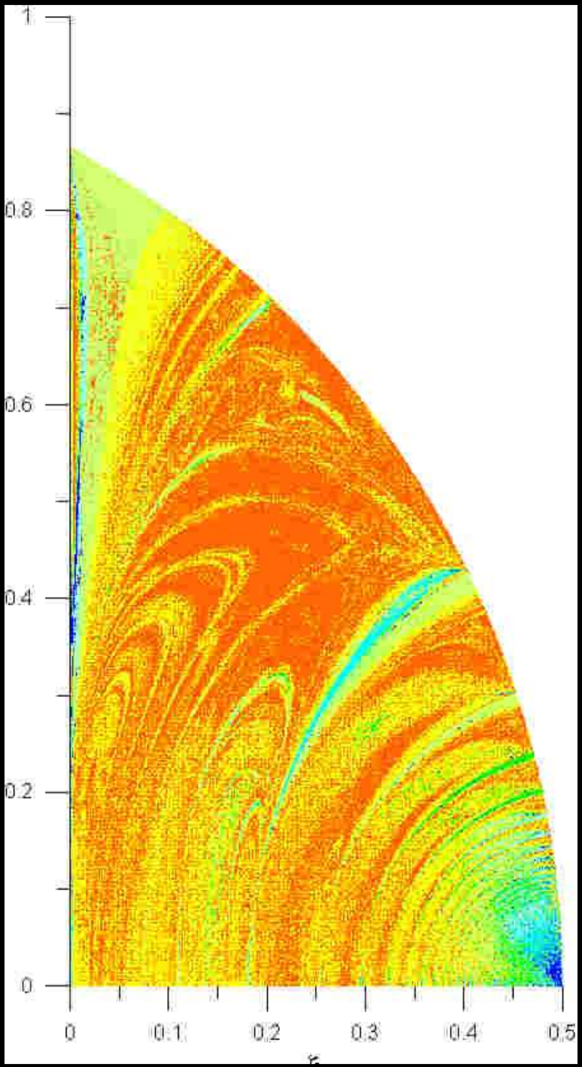
1
0.8
0.6
η
0.4
0.2
0
0
0.1
0.2
0.3
0.4
0.5
ξ
D
.m3

Entropies H1(ξ, η); H2(ξ, η) for subregions of the region D

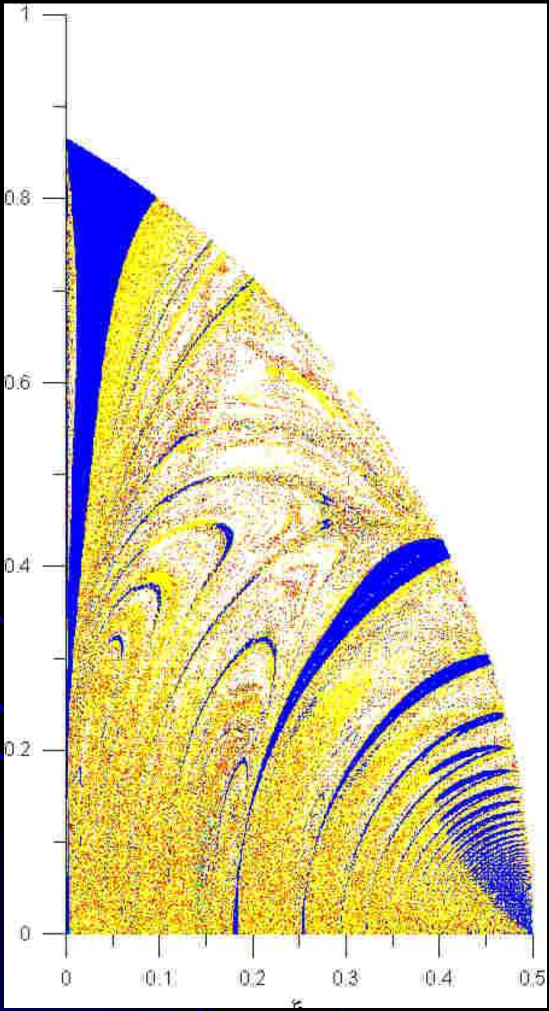


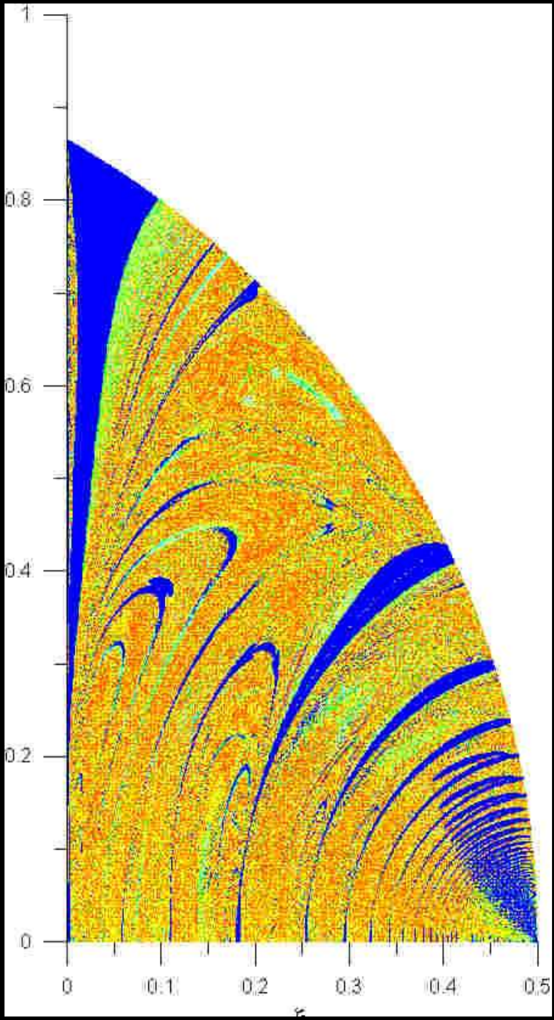
Extreme Values of $H_1(\xi, \eta)$, $H_2(\xi, \eta)$ *for binary encounters*



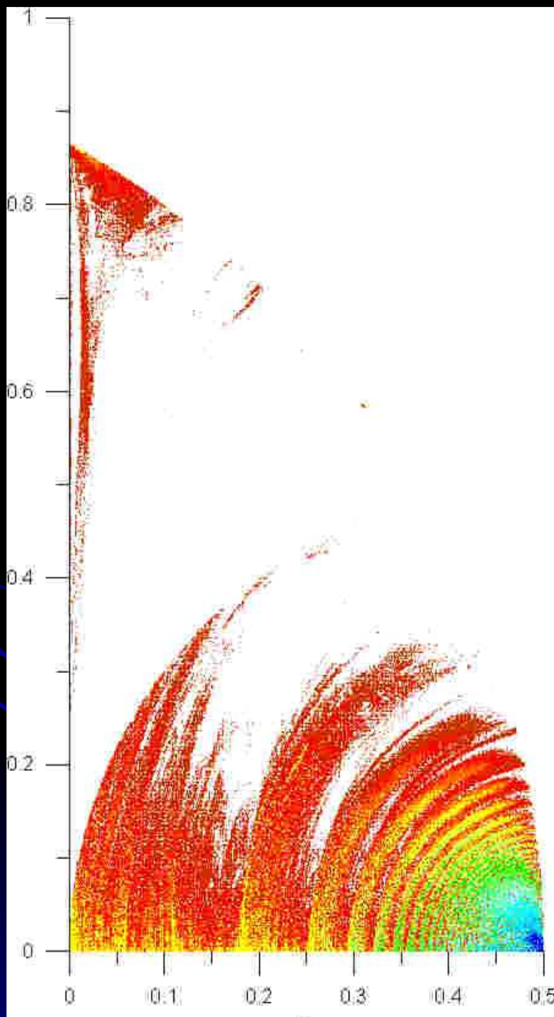


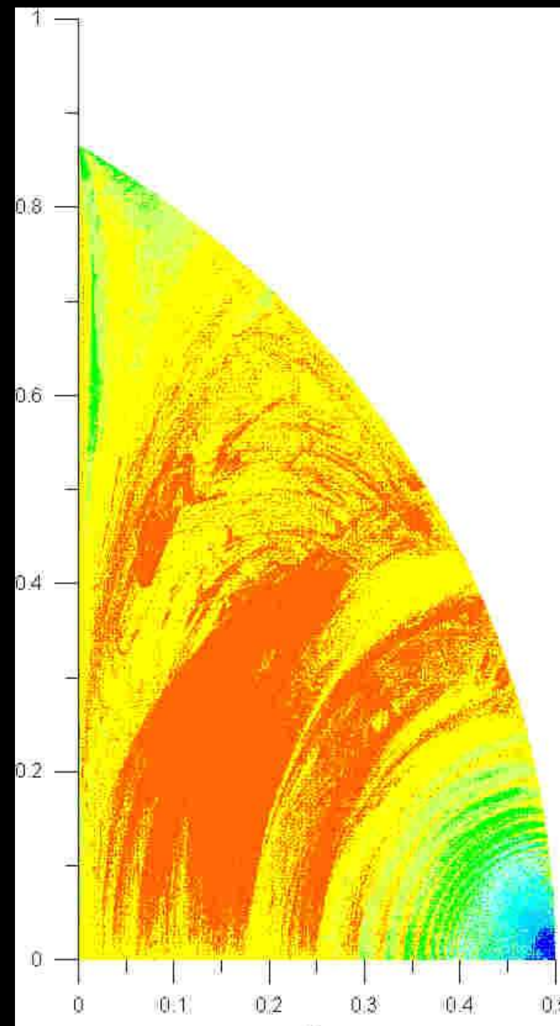
Extreme Values of $H_1(\xi, \eta)$, $H_2(\xi, \eta)$ *for triple encounters*



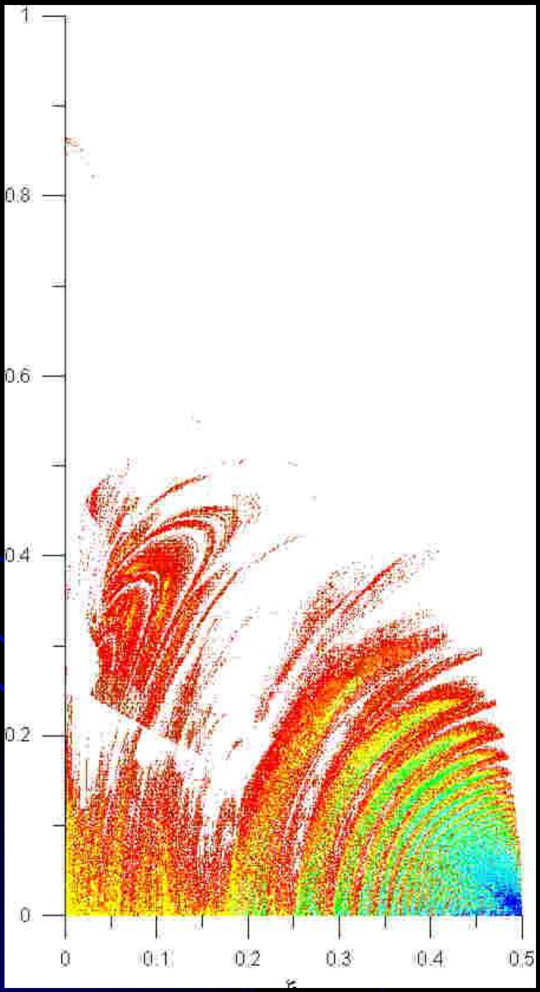


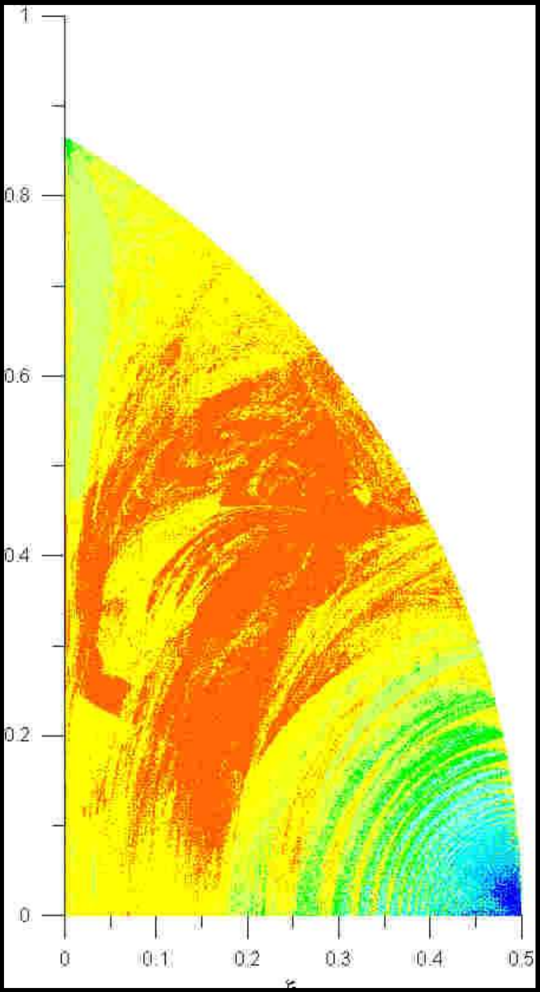
Extreme Values of $H_1(\xi, \eta)$, $H_2(\xi, \eta)$ *for subregions* D_1, D_2, D_3





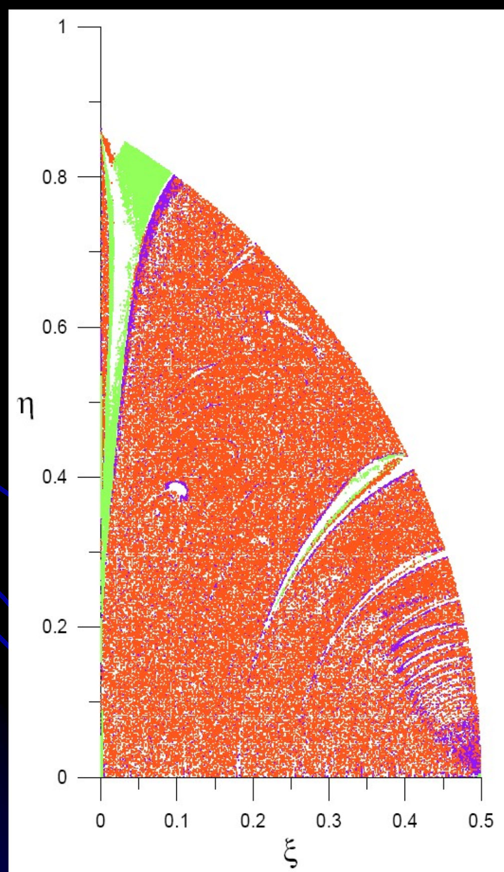
Extreme Values of $H_1(\xi, \eta)$, $H_2(\xi, \eta)$ *for* A, H, L, M

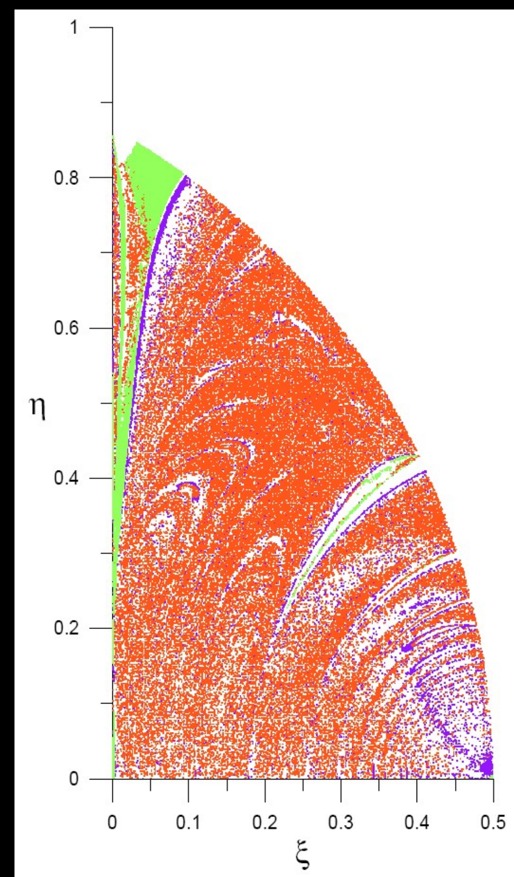




Intermittent Regions of Different Entropies for Binary Encounters

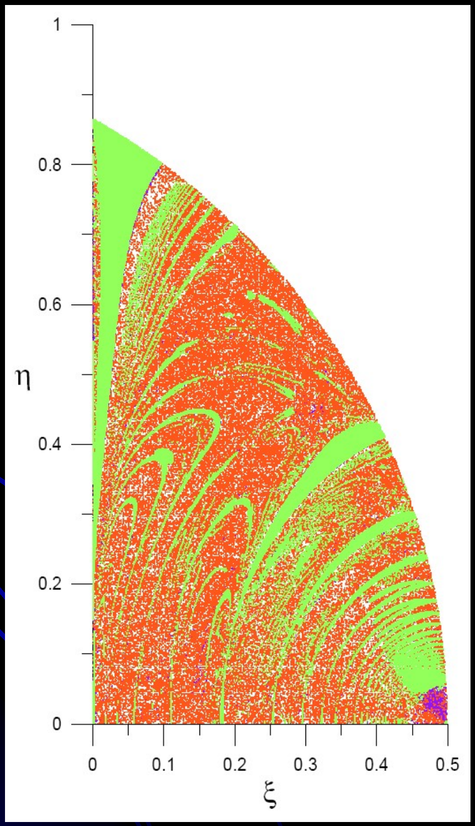
Max $H_{1,2}$ (red) and min $H_{1,2}$ (lilac) and regions of small number of binary encounters n = 1,2,3 (green)

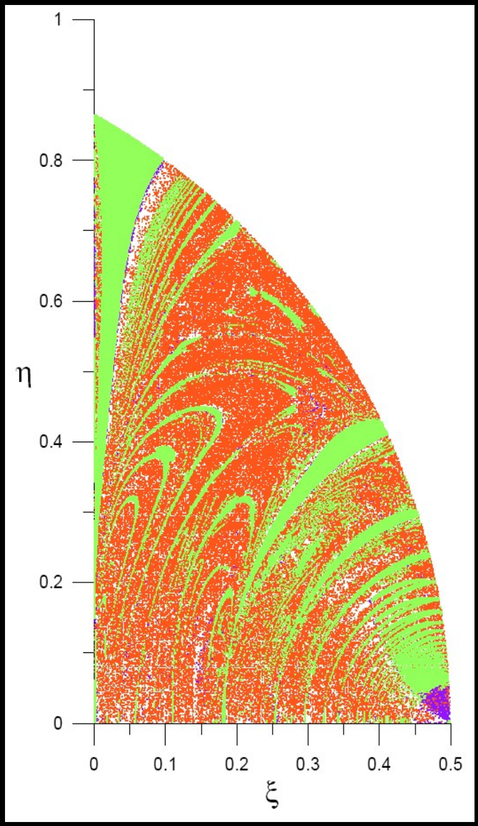




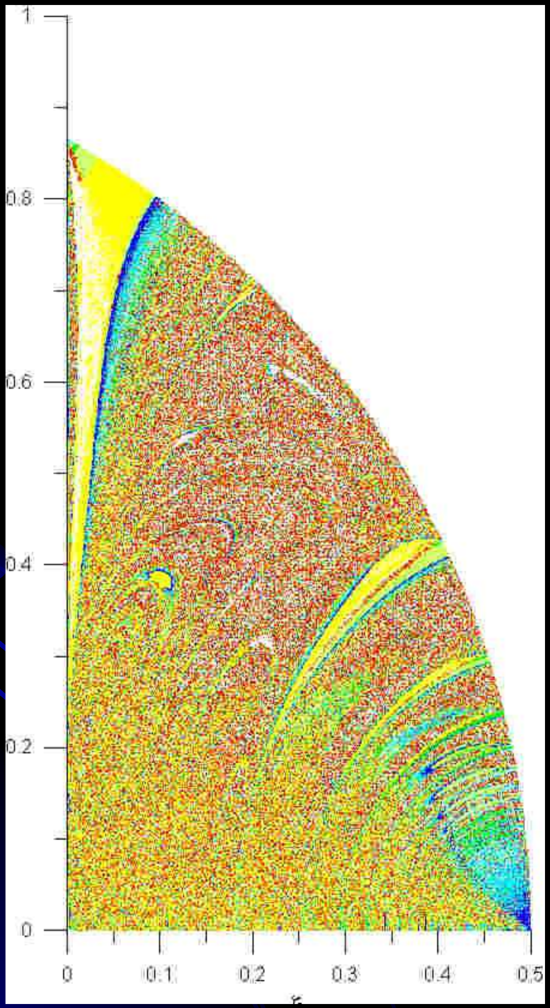
Intermittent Regions of Different Entropies for Triple Encounters

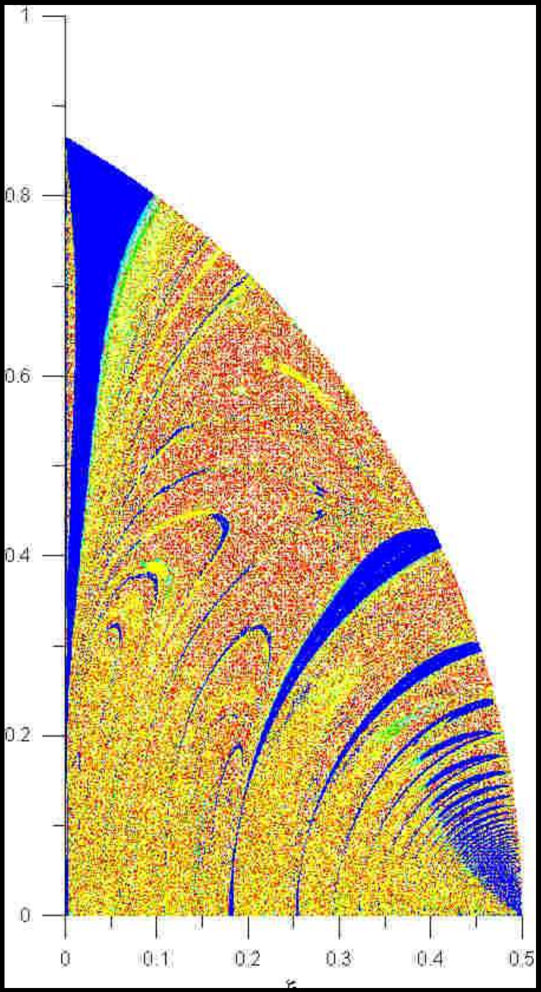
Max $H_{1,2}$ (red) and min $H_{1,2}$ (lilac) and regions of small number of triple encounters n = 1, 2, 3 (green)



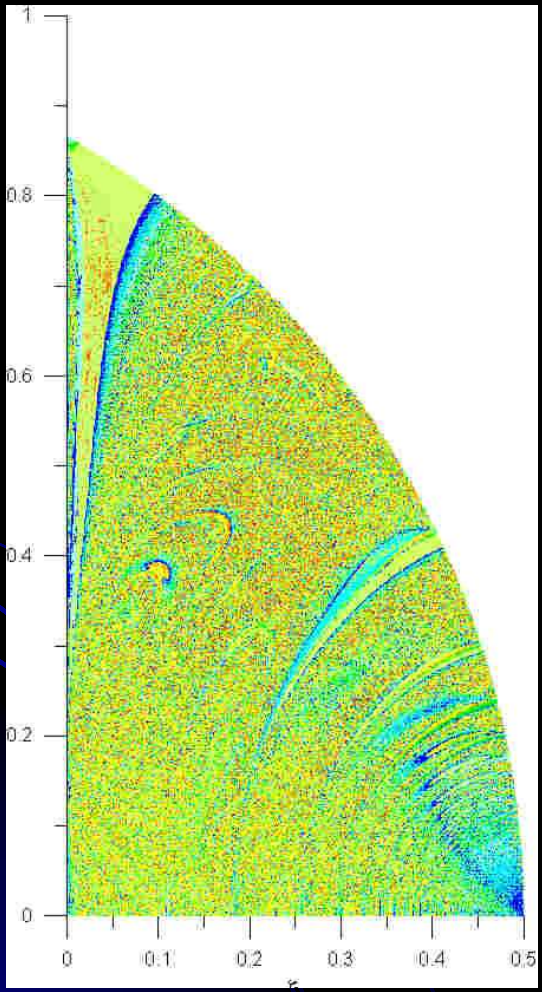


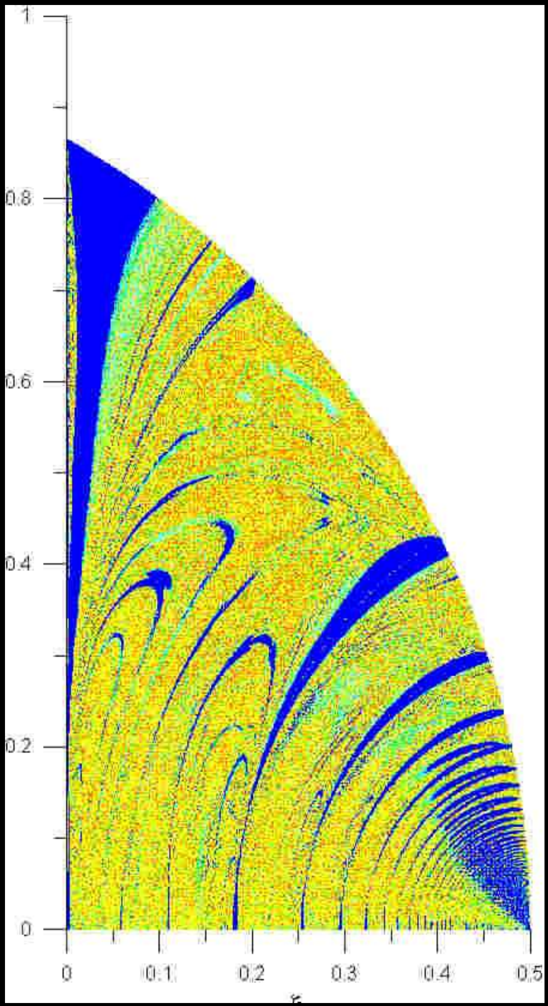
Comparison of Results for Parameter H_1 (binary and triple encounters)



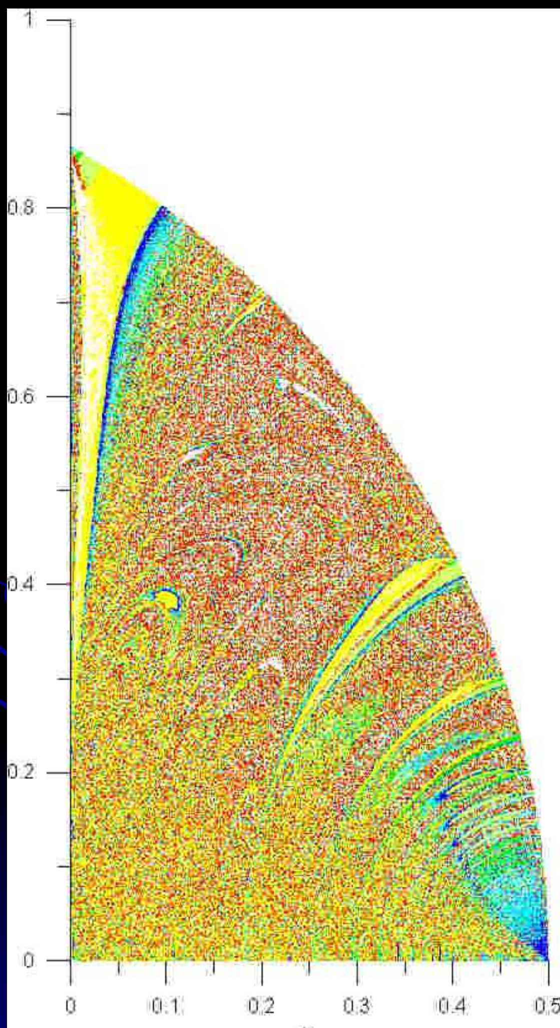


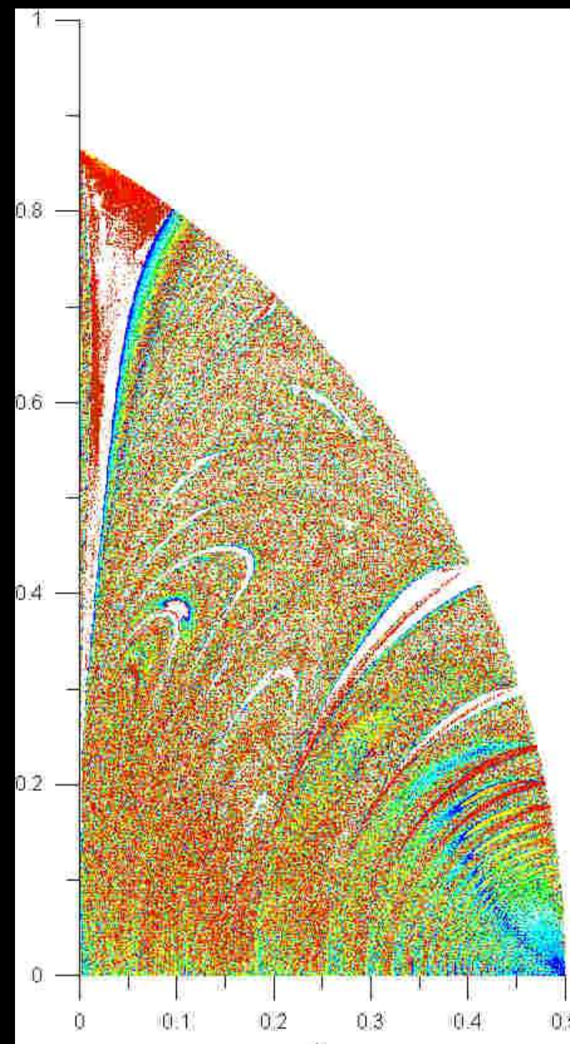
Comparison of Results for Parameter H_2 (binary and triple encounters)



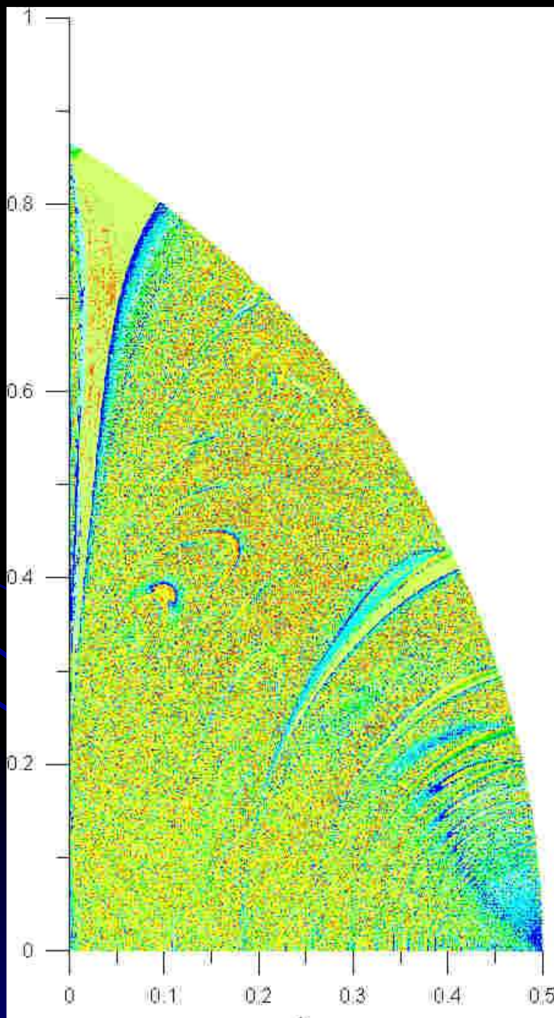


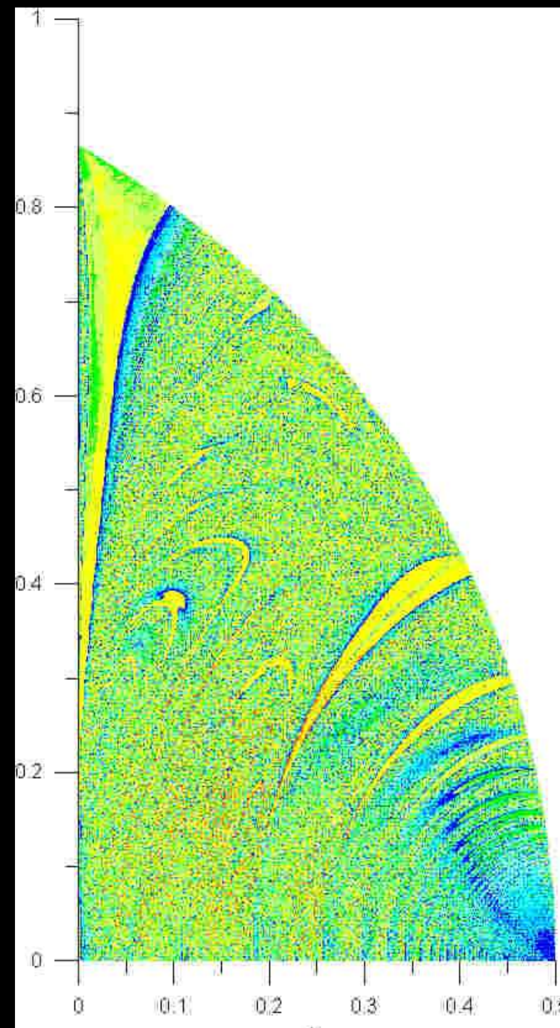
Comparison of Results for Parameter H_1 (binary encounters and subregions D_1, D_2, D_3)





Comparison of Results for Parameter H_2 (binary encounters and subregions D_1, D_2, D_3)





Thank you for attention!